Fuzzy LQR Controller for Heading Control of an Unmanned Surface Vessel

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Abstract— **Recently the applications of unmanned systems are steadily increasing. Unmanned Surface Vessels (USV) can be used for military and rescue purposes. This paper designs a Fuzzy-LQR controller for Heading control of the USV system. A new analysis of the fuzzy system behavior presented helps to make possible precise integration of LQR features into fuzzy control. This Fuzzy-LQR controller is used to adjust the closed loop controller feedback gains in order to obtain the desired Heading (Yaw angle) under variations of the USV parameters and environmental variations. Matlab Simulink has been used to test and compare the performance of the LQR and Fuzzy-LQR controllers. The Fuzzy-LQR controller for USV is verified by simulation to show better performance by suppressing the uncertainty instability more effectively than the LQR besides minimizing the time of the mission proposed.**

***Keywords*—USV, Fuzzy control, LQR control, Heading control**

I. Introduction

Unmanned robotic vessels are capable of performing desired tasks in unstructured, uncertain and potentially hostile environments. They may be remotely-operated or function *(*semi-) autonomously without human intervention. Unmanned surface vessels (USVs) fill an increasingly important niche in the pantheon of robotic vessels. Equipped with appropriate sensors, USVs can collect information about the subsurface and above-surface environments and they can be used for military and rescue purposes.

The ship steering autopilot is one of the earliest applications of automatic control theory. A simplified PID type of course-keeping autopilot was introduced in the early1920s (Minorsky, 1922). Thanks to the deployment of the global positioning system (GPS) in the 1970s, integration of the positional data from the GPS, into ship steering autopilots, forms the so-called track-keeping autopilots. According to the mathematical model, during the controller design process, we can classify the track-keeping autopilot design methods into two categories, the model-based approach and the model-free approach. The LQG optimal control (Holzhuter, 1997) and the HN control (Morawski andPomjrski,1998) are typical model-based methods[1]. The fuzzy control (Vaneck, 1997) and the artificial neural network (ANN) (Hearn etal.,1997) belong to the model-free approach.[2],[3]

The Nomoto model was based off of a four degree of freedom model. In order to simplify the model, the roll motion was neglected. This simplification is acceptable because roll is less significant than yaw, surge, and sway. from this nonlinear three degree of freedom planar motion(surge, sway, and heave) model, with force and moment inputs, we make several simplifying assumptions to obtain low order models suitable for identification from experimental data.[4]

LQR is a method in modern control theory that used state-space approach to analyze such a system. Using state space methods it is relatively simple to work with a multi-output system. The system can be stabilized using full-state feedback system. In designing LQR controller, *lqr* function in Matlab can be used to determine the value of the vector *K* which determined the feedback control law.[5]

Fuzzy Logic Controller (FLC) is conceived as a better method for sorting and handling data but has proven to be an excellent choice for many control system applications because of non-linearity, complex mathematical computation and real-time computation need. [6]

To synthesize a fuzzy controller, we pursue the idea of making it match the LQR for small inputs since the LQR was so successful. Then, we still have the added tuning flexibility with the fuzzy controller to shape its control surface so that for larger inputs it can perform differently from the LQR.[7]

This paper designs a Fuzzy controller for tuning the state feedback gains of the USV system. This Fuzzy-LQR controller is used for heading control of an unmanned surface vehicle. The Fuzzy-LQR simultaneously makes use of the good performance of LQR in the region close to switching curve, and the effectiveness of fuzzy control in region away from switching curve.

The paper is structured as followings: In section 2 the dynamic modelig of the USV is described. The controllers are presented in section 3. The simulations supporting the objectives of the paper and results are presented in section 4.concluding remarks are presented in section 5.

II. Dynamic Modeling of USV

A. *Modeling in Absence of Currents*

With respect to figure 1, the 6 degrees of freedom kinematics of a vessel in the absence of currents can be described by:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{pmatrix}=\begin{pmatrix} c\psi c\theta & -s\psi c\phi+c\psi s\theta s\phi & s\phi s\psi+c\psi c\phi s\theta & 0 & 0 & 0\\ \sigma\psi c\theta & c\psi c\phi+s\psi s\theta s\phi & -c\psi s\phi+s\theta s\psi c\phi & 0 & 0 & 0\\ -s\theta & c\theta s\phi & c\theta c\phi & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & s\phi t\theta & c\phi t\theta\\ 0 & 0 & 0 & 0 & c\phi & -s\phi\\ 0 & 0 & 0 & 0 & s\phi / c\theta & c\phi / c\theta \end{pmatrix}\begin{pmatrix}u\\ v\\ w\\ p\\ q\\ r\end{pmatrix} \quad (1)$$

with c = cos, s = sin, and t = tan.

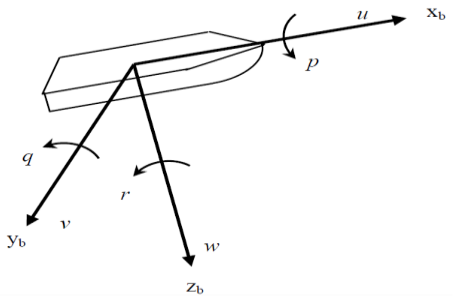

Fig. 1. six DOF vessel model

To simplify matters, we will ignore the roll, pitch, and heave dynamics and consider USV motion in the horizontal plane. This reformulation leads to ignoring planing dynamics, implying that the boat's dynamics cannot be modeled with constant parameters through the entire performance envelope. Different planing situations can be recovered by having different added mass values for each planing mode. Figure 1 illustrates the state and input variables for a planar USV model with a gimbaled thruster (e.g., an outboard engine).[8],[9]

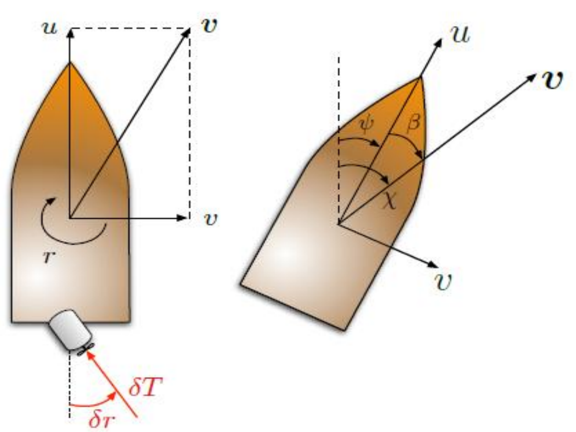

Fig. 2. Three DOF of vessel model

The kinematic model for a USV in planar motion in the absence of currents is:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{pmatrix}=\begin{pmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}u\\ v\\ r\end{pmatrix} \quad (2)$$

B. LOW SPEED NONLINEAR PLANAR MODEL

The dynamics of a boat can be described, using the notation

$$\upsilon=\begin{bmatrix}u & v & r\end{bmatrix}^T, \quad \eta=\begin{bmatrix}x & y & \psi\end{bmatrix}^T \quad (3)$$

as,

$$M\dot{\upsilon}+C(\upsilon)\upsilon+D(\upsilon)\upsilon=f \quad (4)$$

Where M is the mass matrix, C is the Coriolis and centripetal matrix, D is the damping matrix, and f are the control forces and moments. The mass matrix M and Coriolis and centripetal matrix C can be expressed as following relations:

$$M=\begin{pmatrix} m-X_{\dot{u}} & 0 & 0\\ 0 & m-Y_{\dot{\upsilon}} & mx_G-N_{\dot{\upsilon}}\\ 0 & mx_G-N_{\dot{\upsilon}} & I_z-N_{\dot{r}}\end{pmatrix} \quad (5)$$

$$C(\upsilon)=\begin{pmatrix} 0 & 0 & -m(x_Gr+\upsilon)+Y_{\dot{\upsilon}}\upsilon+N_{\dot{\upsilon}}r\\ 0 & 0 & (m-X_{\dot{u}})u\\ m(x_Gr+\upsilon)-Y_{\dot{\upsilon}}\upsilon-N_{\dot{\upsilon}}r & (X_{\dot{u}}-m)u & 0\end{pmatrix} \quad (6)$$

Also assuming planar motion, the linear damping matrix takes the form:

$$D(\upsilon)=\begin{pmatrix} X_u+X_{u|u|}|u| & 0 & 0\\ 0 & Y_\upsilon+Y_{\upsilon|\upsilon|}|\upsilon| & Y_r+Y_{r|r|}|r|\\ 0 & N_\upsilon+N_{\upsilon|\upsilon|}|\upsilon| & N_r+N_{r|r|}|r|\end{pmatrix} \quad (7)$$

At very low speeds, potential fow theory provides a good description of the dominant hydrodynamic effects. Incorporating inputs and decoupled linear damping terms gives:

$$\begin{aligned} m(\dot{u}-vr)&=X_{\dot{u}}\dot{u}-Y_{\dot{v}}vr+X_{u|u|}u|u|+X_{ctrl}(\delta r,\delta \mathrm{T})\\ m(\dot{v}-ur)&=Y_{\dot{v}}\dot{v}+X_{\dot{u}}ur+X_{v|v|}v|v|+Y_{ctrl}(\delta r,\delta \mathrm{T})\\ \mathrm{I}_{zz}\dot{r}&=N_{\dot{r}}\dot{r}+(X_{\dot{u}}-Y_{\dot{v}})uv+N_{r|r|}r|r|+N_{ctrl}(\delta r,\delta \mathrm{T})\end{aligned} \quad (8)$$

In these equations, the propulsor generates a surge force Xctrl which may, in general, be a complicated function of the vehicle state as well as throttle setting $\delta\mathrm{T}$ (measured in percent) and the steering angle δr. Likewise, the steering moment Nctrl($\delta r, \delta\mathrm{T}$) and the side force Yctrl($\delta r, \delta\mathrm{T}$) resulting from control detections may also depend on the full state. The model can be put in the state-space form:

$$\begin{pmatrix}\dot{u}\\ \dot{v}\\ \dot{r}\end{pmatrix}=A\begin{pmatrix}vr\\ ur\\ uv\\ u|u|\\ v|v|\\ r|r|\end{pmatrix}+B\phi_B(\delta r,\delta\mathrm{T}) \quad (9)$$

C. LINEAR NOMOTO MODEL

Linearizing (8) about the steady motion corresponding to the state variable values:

$$u = u_0; v = 0; r = 0$$

The model can be put into state-space form:

$$\begin{pmatrix} \dot{v} \\ \dot{r} \end{pmatrix} = A \begin{pmatrix} v \\ r \end{pmatrix} + B\delta r \quad (10)$$

Nomoto and colleagues [10] suggested that turn rate can be described as a simple 2nd order equation relating rudder angle, δ , and turn rate, r, at a given forward speed u_0 :

$$\frac{r(s)}{\delta(s)} = \frac{K(1+T_3 s)}{(1+T_1 s)(1+T_2 s)} \quad (11)$$

K is a rudder gain and T_i (i=1,2,3) is derived by identification of the vessel.

For the studied USV model the coefficient are derived by system Identification and showed in Table 1.[10],[11]

Table 1
Linear model coefficients

K	0.365 (1/s)
T_1	11.8 (s)
T_2	7.8 (s)
T_3	18.5 (s)

III. HEADING CONTROL ALGORITHM DESIGN

The control is carried out based on the fuzzy model via the so-called parallel distributed compensation scheme.

LQR is used to determine best values for parameters in fuzzy control rules in which the robustness is inherent in the LQR thereby robustness in fuzzy control can be improved. With the aid of LQR, it provides an effective design method of fuzzy control to ensure robustness.

The motivation behind this scheme is to combine the best features of fuzzy control and LQR to achieve rapid and accurate tracking control of a class of nonlinear systems.

The results obtained show a robust and stable behavior when the system is subjected to various initial conditions, moment of inertia and to disturbances.

A. Linear Quadratic Regulator Controller:

The modern control system design is based directly on the state variable model, which contains more information about the system. Another central concept is the expression of performance specifications in terms of a mathematically precise performance criterion that then yields matrix equations for the control gains. The classical successive loop closure approach means that the control gains are selected individually. In contrast, solving matrix equations in modern control allows all the control gains to be computed simultaneously so that all the loops are closed at the same time with stable closed loop poles(as an algorithm condition). This could be achieved by selecting the control input *u* (*t*) to minimize a quadratic cost or performance index (PI) of the type

$$J = \frac{1}{2}\int_0^\infty (\tilde{x}^T Q \tilde{x} + \tilde{u}^T R \tilde{u}) \quad (12)$$

Where ***Q*** and ***R*** are symmetric positive semi-definite weighting matrices $\tilde{x}, \tilde{u}$ are state and control stdeviations respectively.

B. Fuzzy controller

The principles of fuzzy systems are introduced here and then the LQR Integrated fuzzy control is designed .

A fuzzy system comprises a fuzzification unit, a fuzzy rule base, an inference engine and a defuzzification unit. The fuzzy system can be viewed as performing a real (non-fuzzy) and nonlinear mapping from an input vector $X \in R^n$, to an output vector $y = f(X) \in R^m$ (n and m are input and output vector dimensions, respectively). The interfaces between real world and fuzzy world are a fuzzifier and a defuzzifier; the former maps real inputs to their corresponding fuzzy sets and the latter performs in the opposite way to map from fuzzy sets of output variables to the corresponding real outputs. There are two types of fuzzy systems that are commonly used; Takagi-Sugeno-Kang (TSK) and fuzzy systems with fuzzifier and defuzzifier. In this work, we used the second type. The fuzzy rule base consists of fuzzy rules, which use linguistic If-Then sentences to describe the relationship between inputs and outputs.

The antecedent fuzzy set (fuzzy Cartesian product) of each rule $F_1 \times F_2 \times \ldots \times F_n$, is quantified by the t-norm operator which may be defined as (13), the min-operator or the product operator:

$$\mu_{F1\times\ldots\times Fn}(x_1,\ldots,x_n) = \begin{cases} min[\mu_{F1}(x_1),\ldots,\mu_{Fn}(x_n)] \\ or \\ \mu_{F1}(x_1)\ldots\mu_{Fn}(x_n) \end{cases} \quad (13)$$

The defuzzification is performed using (14), where μ_j is the firing strength of the antecedent part of the jth rule and is given by :

$$y = f(X) = \frac{\sum_{j=1}^{N} c_j \mu_j}{\sum_{j=1}^{N} \mu_j} \quad (14)$$

C. The Fuzzy Control LQR (FC-LQR)

The LQR fuzzy control utilizes both advantages from the LQR controller and fuzzy logic controller, as LQR controller can easily satisfy the flying qualities and pilot rating requirements and fuzzy control can cope with the nonlinearity of the system, introducing a smart way to modifying the output gains according to the actual performance, blending the dynamic response that generating better performance than using LQR alone.

A FUZZY-LQR is designed which meets the requirements of small overshoot in the transient response and a well damped oscillations with no steady state error Fig. 3 describes the Fuzzy-LQR controller for yaw angle control. The system dynamics is described by LQR design process for yaw control.

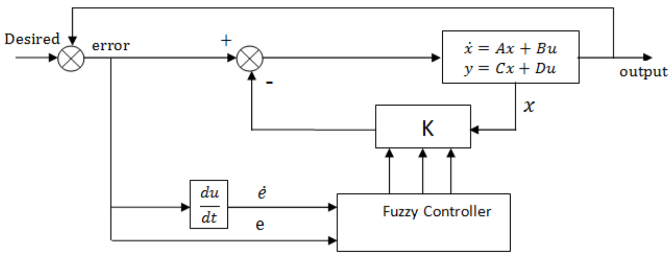

Fig. 2. Fuzzy-LQR controller for yaw angle control

The Fuzzy LQR controller, which takes error "e" and rate of change-in-error "edot" as the input to the controller makes use of the fuzzy controller rules to modify state feedback gains K=[K1,K2,K3] on-line. The Fuzzy LQR controller refers to finding the fuzzy relationship between the three gains of state feedback, K1, K2 and K3 and "e" and "edot", and according to the principle of fuzzy control modifying the three gains in order to meet different requirements for control gains when "e" and "edot" are different and making the control object produce a good dynamic and static performance.

For the input variables of "e" and "edot", seven fuzzy values is chose (NB, NM, NS, ZO, PS, PM, PB) which NB denotes Negative Big, NM denotes Negative Medium, Negative Small (NS) , Zero (ZO) , Positive Small (PS), Positive Medium (PM) and PB denotes Positive Big, and for the outputs we chose nine fuzzy values (NVB, NB, NM, NS, Z, PS, PM,PB,PVB) which NVB denotes negative Very large and PVB denotes Positive Very Big.

The membership functions of all the outputs have been chosen identical. Fig. 4, Fig. 5 shows these membership functions. This membership functions are combined of triangular and Gaussian. The width of the fuzzy sets used for controllers are not same and they have been determined by trial and error experience. The fuzzy sets width of outputs has been chosen [-500,500]. for inputs, the range of the error was chosen [-1 1] and for error rate have been chosen [-10 10].

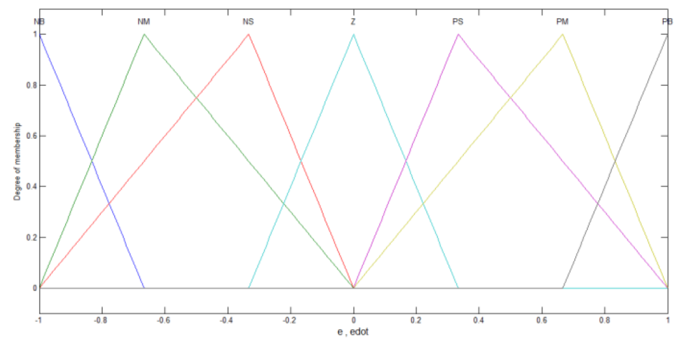

Fig.4. Membership function of Inputs

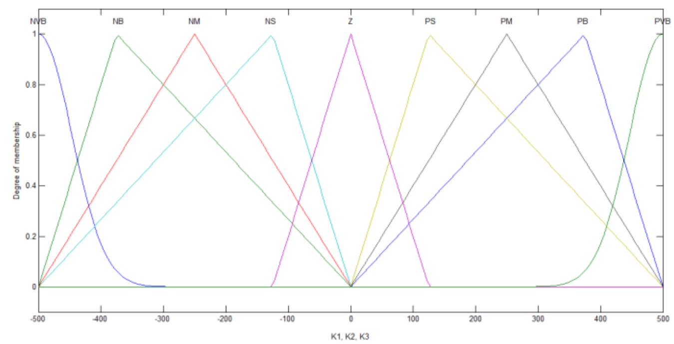

Fig. 3. Membership function of Outputs

IV. *RESULTS AND SIMULATION STUDY*

In this section, we will present simulation results of the proposed fuzzy LQR controller for yaw control of USV and compare them with the LQR's.The mathematical dynamical model of the Unmanned Surface Vessel as well as the controllers have been developed in Matlab Simulink for simulation.

The simple LQR controller gains was computed by the lqr function of MATLAB, and the appropriate Q and R matrices were calculated by trial and error approach. The resulting gain is: K=[-316.2 -411.8 -265.4].

The step response of two controllers are compared and presented in Fig. 6. Dotted black line is the input, dashed red line is LQR response and Solid blue line is Fuzzy-LQR response. A noticeable decrease in overshoot is seen in Fuzzy-LQR controller and the settling time decreased in Fuzzy-LQR procedure. Also the oscillating nature of LQR response in completely removed in Fuzzy-LQR response. Clearly the Fuzzy-LQR controller shows better results than simple tuned LQR.

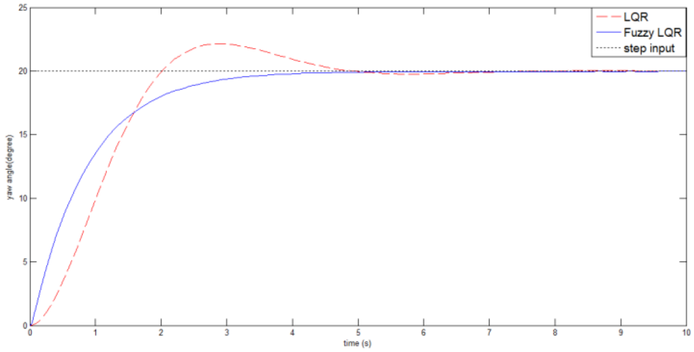

Fig. 4. Step response of USV

For the tracking problem we inserted the sine wave as input and the responses of controllers are shown in Fig. 7. . Dotted black line is the input, dashed red line is LQR response and Solid blue line is Fuzzy-LQR response. Obviously the Fuzzy-LQR controller tracked the input displacement better, and this controller tracks the desired path more rapidly.

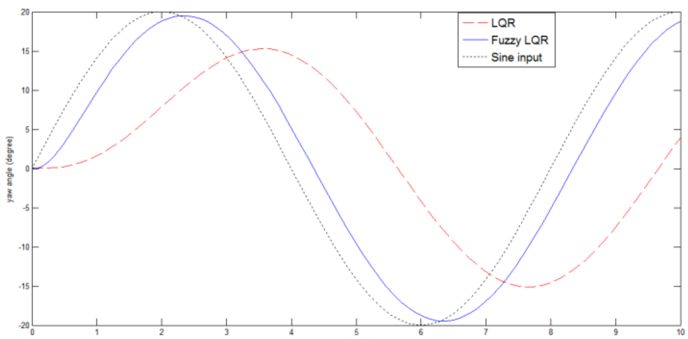

Fig. 5. Tracking response for USV

V. *CONCLUSIONS*

In this paper we first derived the low speed nonlinear planar model of a vessel. Since the vessel's nonlinearities and the variations in environmental forces, the conventional controllers show poor performance. We investigate a new procedure to overcome this problem.

Linear Quadratic Regulator (LQR) is modern linear control that is suitable for multivariable state feedback and is known to yield good performance for linear systems. The fuzzy control is known to have the ability to deal with nonlinearities without having to use advanced mathematics.

By blending these two controllers, a Fuzzy-LQR controller for the Heading control of an USV has been designed. The main idea is to design a supervisory fuzzy controller capable to adjust the closed loop controller feedback gains in order to obtain the desired Heading (Yaw angle) under variations of the USV parameters and environmental variations. The motivation behind this scheme is to combine the best features of fuzzy control and that of the optimal LQR.

Various simulations based on the Matlab Simulink, were performed to test and compare the LQR and Fuzzy-LQR controllers. The weighting matrices of LQR are calculated by trial and error approach to achieve the best performance.

The designed Fuzzy-LQR adjusts the feedback gains based on rule-base which get error and error rate as input. Performance of Designed Fuzzy-LQR controller is superior to LQR. It tracks the desired path rapidly and more precisely than conventional controllers. Therefore the designed USV model and controller will be useful for advanced real embedded platform.

VI. REFERENCES

[1] T. Holzhuter, "LQG approach for the high-precision track control of ships," *Inst. Electr. Eng. Proc. Control Theory Appl.*, vol. 144, no. 2, pp. 121–127, 1997.

[2] T. W. Vaneck, "Fuzzy guidance controller for an autonomous boat," *IEEE Control Syst. Mag.*, vol. 17, no. 2, pp. 43–51, Apr. 1997.

[3] R. Skjetne, T. I. Fossen, and P. V. Kokotovic, "Adaptive maneuvering with experiments for a model ship in a marine control laboratory," *Automatica*, vol. 41, no. 2, pp. 289–298, 2005.

[4] C. Tzeng and J. Chen, "Fundamental Properties of Linear Ship Steering Dynamic Models," Journal of Marine Science and Technology, vol. 7, no. 2, pp. 79-88, 1999.

[5] R.G. Berstecher, R. Palm, H.D. Unbehauen, An adaptive fuzzy sliding
mode controller, IEEE Trans. Indust. Electron. 8 (1) (2001) 18U″ 31.
1024U″ 1030.

[6]Sugeno M, Yasukawa T. A fuzzy-logic-based approach to qualitative modeling[J]. IEEE Transaction Fuzzy System, 1993, 1(1): 7-31.

[7] Kevin, M, Passino and Yarkovich, S., FUZZY CONTROL , Department of Electrical Engineering, Ohio State University.

[8] Volker Bertram. Unmanned surface vehicles - a survey.

[9] M. Breivik, V. E. Hovstein, and T. I. Fossen. Straight-line target tracking for unmanned surface vehicles. Modeling, Identi_cation, and Control, 29(4):131{149, 2008.

[10] O. M. Faltinsen. Hydrodynamics of High-Speed Marine Vehicles. Cambridge University Press, 2005.

[11] T. I. Fossen. Guidance and Control of Ocean Vehicles. John Wiley and Sons, 1995.