Adaptive Neuro-Fuzzy control of an unmanned bicycle

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***Abstract*— In this work, an adaptive critic-based neuro-fuzzy is presented for an unmanned bicycle. The only information available for the critic agent is the system feedback which is interpreted as the last action the controller has performed in the previous state. The signal produced by the critic agent is used alongside the back propagation of error algorithm to tune online conclusion parts of the fuzzy inference rules. Stability and roll angle tracking controls for an unmanned bicycle are presented. The effectiveness of the control schemes is proved by simulation results.**

Keywords— Adaptive Neuro-fuzzy controller, unmanned bicycle, Stability, Roll angle tracking.

I. INTRODUCTION

There are several reasons which show the importance of unmanned bicycle Stabilization, firstly, the bicycle has very complicated dynamic and nonlinear differential equation with non-holonomic constrain conditions, so it is great to examine the controller efficiency for the similar problems which have the same degree of complexity. Secondly, it’s useful for rehabilitating with bicycle simulator devices which help crippled people. Lastly, it can be used in bicycle robots and stabilize it to track a path.

Modeling and control of bicycles became a popular topic for researchers in the late half of the last century. During the early of 20th century, several authors studied the problems of self-stabilization, balancing, and steering.

Yamakita et al. utilized an input-output linearization method to design a trajectory tracking controller for an automatic bicycle [1] and [2].

H. D. Sharna et al. introduced an intelligent controller for stabilizing an autonomous bicycle system. The controller was developed by using fuzzy logic approach which the rule set was designed by using of the inherent-characteristic relationship of lean and steer present in a bicycle [3]. C. K. Chen and T.S Doa presented a steady turning and roll angle tracking control for unmanned bicycle with fuzzy controller by fixed parameters and rules [4]. Later they designed a Genetic fuzzy control for path tracking of an autonomous bicycle which optimized the membership function parameters with Genetic Algorithm (GA) [5].

A. Fakhrazari and M Boroushaki presented an adaptive critic-based neuro-fuzzy controller for the steam generator water level [6]. The paper focused on fuzzy critic-based learning and its reinforcement learning method is based on dynamic programming. The proposed controller was optimized by gradient descent Algorithm which shows satisfactory transient responses, disturbance rejection and robustness to model uncertainty.

II. PROBLEM STATEMENT

A. Bicycle Model

The mechanical model of the bicycle consists of four rigid bodies, viz. the rear frame with the rider rigidly attached to it, the front frame being the front fork and handle bar assembly and the two knife-edge wheels. These bodies are interconnected by revolute hinges at the steering head between the rear frame and the front frame and at the two wheel hubs. In the reference configuration, all bodies are symmetric relative to the bicycle mid plane. The contact between the stiff non-slipping wheels and the flat level surface is modeled by holonomic constraints in the normal direction and by non-holonomic constraints in the longitudinal and lateral direction. There is no friction, apart from the idealized friction between the non-slipping wheels and the surface, with propulsion and rider control. In the reference position, the global Cartesian coordinate system is located at the rear-wheel contact point *O*, where the x-axis points in the longitudinal direction of the bicycle and the z-axis is directed downwards. Fig.1 shows the directions of the axes, where the terminology used mainly follows the SAE recommended practice as described in the report SAE-J607e [SAE, 2001], last revised in 1976.

The mechanical model of the bicycle has three degrees of freedom: the roll angle φ of the rear frame, the steering angle δ, and the rotation θ_r of the rear wheel with respect to the rear frame. The angles are defined as follows. The orientation of the rear frame with respect to the global reference frame *O–xyz* is given by a sequence of three angular rotations: a yaw rotation, ψ, about the *z*–axis, a roll rotation, φ, about the rotated *x*– axis, and a pitch rotation, θ, about the rotated *y*–axis.

The steering angle δ is the rotation of the front frame with respect to the rear frame about the steering axis. The four kinematic coordinates are taken here as the Cartesian coordinates x and y of the rear-wheel contact point, the yaw angle ψ of the rear frame, and the rotation θ_f of the front wheel with respect to the front frame.

The dimensions and mechanical properties of the benchmark model are presented in Table 1. The system is symmetric about the vertical longitudinal plane and the wheels are rotationally symmetric about their axles. The mass moments of inertia are given at the centre of mass of the individual bodies and along the global xyz-axes.

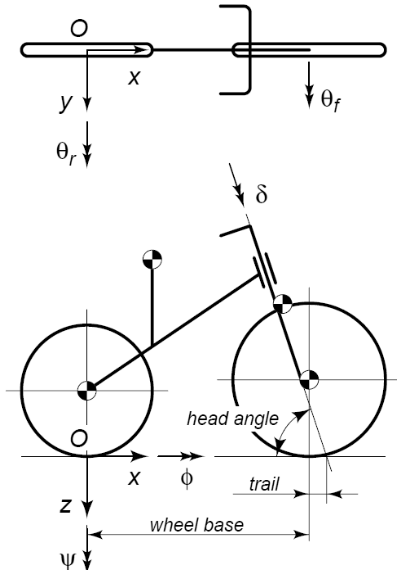

Fig. 1. Bicycle model together with the cordinate system, the degrees of freedom and the parameters.

B. Linearized Equations of Motion

The equations of motion are obtained by pencil-and-paper using D'Alembert's principle and linear and angular momentum balances. They are expressed in terms of small changes in the degrees of freedom φ, the rear frame roll angle, and δ, the steering angle, from the upright straight ahead configuration $\varphi_0 = 0$, $\delta_0 = 0$, at a forward speed of $v = -R\,\dot{\theta}_r$.

The linearized equations of motion for the bicycle expressed in the degrees of freedom qd=(φ,δ)T have the form:

$$M\ddot{q}^d + \left[C_1 v\right]\dot{q}^d + \left[K_0 + K_2 v^2\right]q^d = f^d \qquad (1)$$

With a constant mass matrix, M, a "damping" matrix C_1. v which is linear in the forward speed, and a stiffness matrix which is the sum of a constant part, K_0, and a part, $K_2.v^2$, which is quadratic in the forward speed. The linearized equation of motion for the third degree of freedom, the rotation θ_r of the rear wheel, is decoupled from the first two (1) and takes on the very simple form of: $\ddot{\theta}_r = 0$. This means that the forward speed remains constant for small changes in the upright configuration. The forces on the right hand side, fd, are the applied forces which are energetically dual to the degrees of freedom qd. For the bicycle model the first is Mφ, the action-reaction roll moment between the fixed space and the rear frame. In practice such a torque could be applied by side wind, or by a parent teaching a child to ride. The second is Mδ, the action-reaction steering moment between the rear frame and the front frame. This is the torque that would be applied by a rider's hands, a steering spring damper, or a controller.

Substitution of the parameter values from Table 1 results in the following values for the entries in the mass matrix, the "damping" matrix, the constant stiffness matrix and the stiffness matrix which is proportional to the square of the forward speed froms:

$$\boldsymbol{M} = \begin{bmatrix} 80.812 & 2.323 \\ 2.323 & 0.301 \end{bmatrix} \quad ,\boldsymbol{C}_1 = \begin{bmatrix} 0 & 33.774 \\ -0.848 & 1.707 \end{bmatrix} \qquad (2)$$

$$\boldsymbol{K}_0 = \begin{bmatrix} -794.119 & -25.739 \\ -25.739 & -8.139 \end{bmatrix} \ , \boldsymbol{K}_2 = \begin{bmatrix} 0 & 76.406 \\ 0 & 2.675 \end{bmatrix} \qquad (3)$$

Table 1 Parameters for the benchmark bicycle

Parameter	**Value**
Wheel base	1.02 m
Trail	0.08 m
Head angle	arctan(3)
Gravity	9.81 N/kg
Forward speed	Variable m/s
Rear wheel	
Radius	0.3 m
Mass	2 kg
Mass moments of inertia	(0.06,0.12,0.06) kgm^2
Rear frame	
Position centre of mass	(0.3,0,-0.9) m
Mass	85 kg
Mass moments of inertia	$\begin{bmatrix} 9.2 & 0 & 2.4 \\ 0 & 11 & 0 \\ 2.4 & 0 & 2.8 \end{bmatrix}$ $kg.m^2$
Front frame	
Position centre of mass	(0.9,0,-0.7) m
Mass	4 kg
Mass moments of inertia	$\begin{bmatrix} 546 & 0 & -162 \\ 0 & 600 & 0 \\ -162 & 0 & 114 \end{bmatrix}$ $*10^{-4}$ $kg.m^2$
Front Wheel	
Radius	0.35 m
Mass	3 kg
Mass moments of inertia	(0.14,0.28,0.14) kgm^2

III. ADAPTIVE CRITIC-BASED NEUROFUZZY CONTROLLER

A. Neuro-fuzzy Networks

In this subsection, the principles of fuzzy systems are introduced and then an equivalent architecture that incorporates fuzzy system concept into an adaptive neural network concept, is obtained, hence the name neurofuzzy.

In general, a fuzzy system comprises a fuzzification unit, a fuzzy rule base, an inference engine and a defuzzification unit. The fuzzy system can be viewed as performing a real (non-fuzzy) and nonlinear mapping from an input vector $X \in R^n$, to an output vector $y = f(X) \in R^m$ (n and m are input and output vector dimensions, respectively). The interfaces between real world and fuzzy world are a fuzzifier and a defuzzifier; the former maps real inputs to their corresponding fuzzy sets and the latter performs in the opposite way to map from fuzzy sets of output variables to the corresponding real outputs. There are two types of fuzzy systems that are commonly used; Takagi-Sugeno-Kang (TSK) and fuzzy systems with fuzzifier and defuzzifier. In this work, we used the first type. The fuzzy rule base consists of fuzzy rules, which use linguistic If-Then sentences to describe the relationship between inputs and outputs.

Consider a Multiple-Input Single-Output (MISO) fuzzy system consisting of N rules as follows:

Rj: If (x1 is Fj1) and … and (xn is Fjn) Then cj=gj(X)

Where j=1,2,…,N; xi(i=1,2..,n) are the input variables of the fuzzy system, Fji is characterized by its corresponding membership function μFji(xi), cj is the consequence of the jth rule.

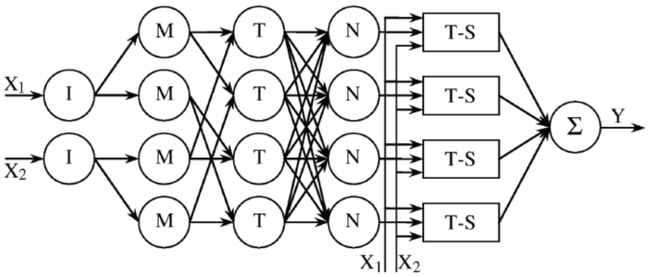

Fig. 2. A sample neurofuzzy structure equivalent with a MISO TSK fuzzy inference system

The antecedent fuzzy set (fuzzy Cartesian product) of each rule $F_1 \times F_2 \times \ldots \times F_n$, is quantified by the -norm operator which may be defined as (4), the min-operator or the product operator:

$$\mu_{F1\times\ldots\times Fn}\left(x_1,\ldots,x_n\right)=\begin{cases} min[\mu_{F1}(x_1),\ldots,\mu_{Fn}(x_n)] \\ or \\ \mu_{F1}(x_1)\ldots\mu_{Fn}(x_n) \end{cases} \tag{4}$$

The defuzzification is performed using (5), where μj is the firing strength of the antecedent part of the jth rule and is given by (6)

$$y=f\ (X\ )=\frac{\sum_{j=1}^{N}c_j\mu_j}{\sum_{j=1}^{N}\mu_j} \tag{5}$$

$$\mu_{F1}(x_1)\ldots\mu_{Fn}(x_n) \tag{6}$$

In TSK fuzzy systems, the consequent part of rules is given by (7)

$$c_j=a_{0j}+\sum_{i=1}^{n}a_{ij}.x_i \tag{7}$$

Where a_{0j} and aijs are the coefficients that should be set at design stage or tuned during the corresponding learning procedure.

Implementing a fuzzy inference system in the framework of an adaptive neural network results in a six layer network in which each layer serves as one part of the equivalent fuzzy system. Fig. 2 shows a sample neuro-fuzzy system equivalent with a two-input and one-output TSK fuzzy inference system which has two linguistic labels for each input and therefore four rules in its rule base.

The first layer nodes specified by I, assign input scaling factors in order to map inputs to [-1,+1] range. Each node in the second layer denoted by M specifies the degree to which the given input satisfies the linguistic label, thus calculating $\mu_{Fji}(x_i)$. Third layer nodes denoted by T, multiply the incoming signals and constitute the antecedent parts of fuzzy rules, $\mu_{F1}(x_1)\ldots\mu_{Fn}(x_n)$ (multiplication implies choosing the product-operator for the t-norm operator). Each node in the forth layer specified by N, calculates the ratio of corresponding firing strength to the sum of all rules firing strengths, hence the term $\mu_j/\sum\mu_j$. The function of nodes in the fifth layer is performing a linear combination on inputs and adding a constant value, thus calculating the corresponding rule's consequent part c_j. T-S labels on Fig. 2 refer to TSK rules. The coefficients of these linear combinations and that of constant value will be adapted during the learning stage. Finally, in the last layer, acting as the defuzzifier, the output is obtained and is according to (5).

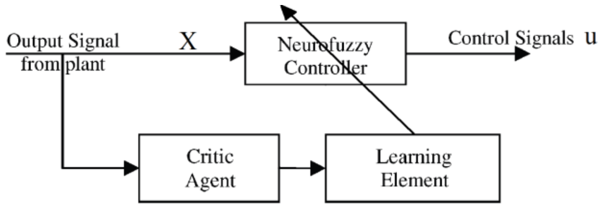

Fig. 3. Structure of adaptive critic-based neurofuzzy controller(ACNFC)

B. *The Controller Structure*

The critic agent assesses the controller performance through evaluation of plant output and provides appropriate reinforcement signal. The signal produced, contributes collaboratively for updating parameters of the neuro-fuzzy controller.

Let define the error function E as

$$E=\frac{1}{2}r^2 \tag{8}$$

The goal of the learning procedure is minimization of E, so the tunable parameters should be updated in the opposite direction of $\overrightarrow{\nabla E}$ (∇ is the gradient operator). This can be stated as follows:

$$\Delta\omega \propto -\frac{\partial E}{\partial \omega} \tag{9}$$

where ω is the tunable parameter of the neuro-fuzzy controller. Equation (9) is in fact the steepest decent law. Applying the chain rule for calculating the relative derivative of (9) we have:

$$\frac{\partial E}{\partial \omega}=\frac{\partial E}{\partial r}\times\frac{\partial r}{\partial u}\times\frac{\partial u}{\partial \omega} \tag{10}$$

where u is the control signal.

Let define the reinforcement signal as a linear combination of error e=y_{ref}-y, and derivative of error $\dot{e}$ as in (9), where y_{ref} and y are the reference and actual outputs of the plant under control, respectively.

$$r = k_1 e + k_2 \dot{e} \tag{11}$$

Where k_1 and k_2 are positive constants.

Applying the chain rule and using (9) we can write

$$\frac{\partial r}{\partial u} = \frac{\partial r}{\partial y} \times \frac{\partial y}{\partial u} = \left(\frac{\partial r}{\partial e} \times \frac{\partial e}{\partial y}\right) \times \frac{\partial y}{\partial u} = (-1 \times k_1) \times \frac{\partial y}{\partial u} \tag{12}$$

$$\frac{\partial E}{\partial \omega} = r \times (-k_1 \times \frac{\partial y}{\partial u}) \times \frac{\partial u}{\partial \omega} \tag{13}$$

In (13), the term ∂y/∂u is the gradient of the system and shows the long term variations of the plant output to the control signal. As in most cases, the system is designed in such a way that this variation is a positive constant.

Using (9) and (13), adaptation rule of the tunable parameter will be as follows:

$$\Delta\omega = \eta \times r \times \frac{\partial u}{\partial \omega} \tag{14}$$

Where η>0 is the learning rate parameter which embeds the proportionality constant of (9) as well as the constant values of (13).

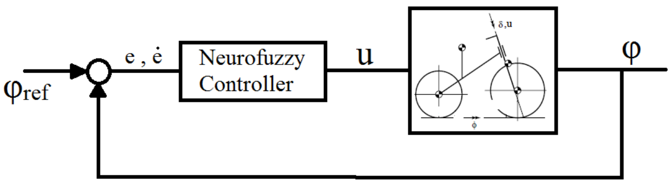

Fig. 4. Structure of ACNFC for bicycle

For the neuro-fuzzy controller introduced in the previous subsection, the control signal using (5) and (7) has the form as in (15)

$$u = \frac{\sum_{j=1}^{N} (a_{0j} + \sum_{i=1}^{n} a_{ij} . x_i) \mu_j}{\sum_{j=1}^{N} \mu_j} \tag{15}$$

Hence, in according to (14) the update rules for the parameters of the neuro-fuzzy controller will be given as (16) and (17)

$$\Delta a_{0j} = \eta \times r \times \frac{\mu_j}{\sum_{j=1}^{N} \mu_j} \tag{16}$$

$$\Delta a_{ij} = \eta \times r \times x_i \times \frac{\mu_j}{\sum_{j=1}^{N} \mu_j} \tag{17}$$

C. Controller design for the bicycle

Structure of the adaptive critic-based neuro-fuzzy controller (ACNFC) used for the stability control of the bicycle is shown in Fig. 4.

The neuro-fuzzy controller element of ACNFC, see Fig. 2, uses roll angle error (e_φ) and its derivative ($\dot{e}_\varphi$) as inputs and has three linguistic variables, i.e., Negative (N), Zero (Z) and Positive (P) thus includes nine rules in its rule base. The membership function of the linguistic variable, Z, is chosen as the Gaussian function and that of linguistic variables N and P are chosen as the Sigmoid function. The membership functions of the linguistic variables are shown in Fig. 5.

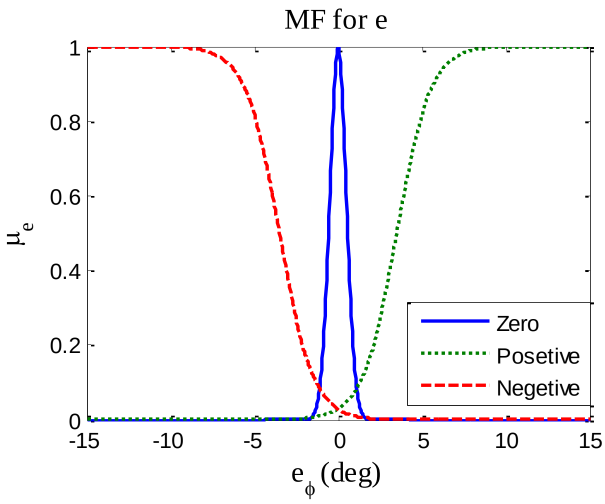

Fig. 5. The membership function of the corresponding linguistic variables

As the neuro-fuzzy controller has error and derivative of error of the roll angle as inputs and there are nine rules in the fuzzy rule base, using (15) and ignoring bias term (a_{0j}) the parameters of the neuro-fuzzy controller will be updated as follows:

$$c_j = a_{1j} e_\varphi + a_{2j} \dot{e}_\varphi \tag{18}$$

$$\Delta a_{1j} = \eta \times r \times e_\varphi \times \frac{\mu_j}{\sum_{j=1}^{N} \mu_j}, j = 1, 2, .., 9 \tag{19}$$

$$\Delta a_{2j} = \eta \times r \times \dot{e}_\varphi \times \frac{\mu_j}{\sum_{j=1}^{N} \mu_j}, j = 1, 2, \ldots, 9 \tag{20}$$

Where η=0.001

IV. SIMULATION RESULTS

In this section, we will present simulation results of the proposed adaptive critic-based neuro-fuzzy controller.

As the bicycle is unstable at low velocities, the main goal is to obtain a controller which stabilize bicycle at low velocities. With that in mind, for comparing the neuro-fuzzy controller performance the PD controller was introduced due to neuro-fuzzy inputs (error and error rate) and it was tuned for v=1m/s. the performance of bicycle and controllers for initial condition (initial roll angle is about 6 deg) without any external disturbances were depicted in Fig.6 which shows the neuro-fuzzy controller needs less control effort than PD controller.

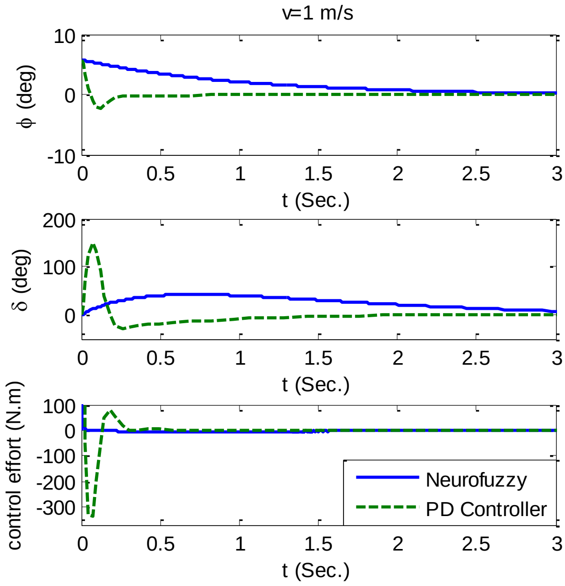

Fig. 6. Initial Response of Bicycle at v=1m/s a) Roll angle, b) Steer angle and c) control effort

According to use linearized equation of motion which is true only for constant speed, the performance of designed controller should be checked for different velocities. Fig.7 compare robustness of neuro-fuzzy and PD controller. The PD Controller which was tuned at 1m/s velocity is used for stabilizing bicycle at 5m/s velocity. From Fig.7 robustness of neuro-fuzzy is obvious.

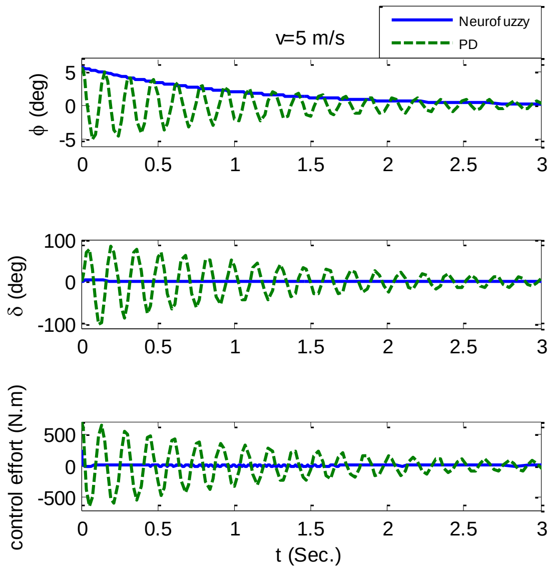

Fig. 7. Initial Response of Bicycle at v=5m/s a) Roll angle, b) Steer angle, c) Control effort

Due to its highly nonlinear non-holonomic dynamics and instability, the autonomous bicycle possesses several properties making it difficult for path-tracking. Unlike other mobile robotic systems, it is unable to control the steering angle or change the orientation of the bicycle directly to follow a given path because that makes the vehicle fall down. In this study, a control scheme is proposed to control the steering angle indirectly by changing its roll-angle. So for path planing we need roll angle tracking.

Fig. 8 shows a simulation in which the bicycle is controlled to follow a desired sinusoidal roll angle φ(t)=10sin(0.5t). The result indicates the effectiveness of the neurofuzzy for controlling the unmanned bicycle to follow a time-varying roll angle. as shown in Fig.8 initial roll angle of bicycle is about 6^0 which is comes to desired roll angle less than 2 seconds.

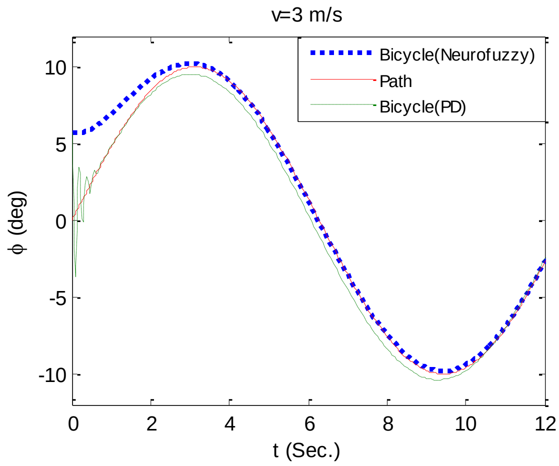

Fig. 8. Roll angle Tracking Performance of PD and Neurofuzzy Controller

V. CONCLUSIONS

In this study, a linearized dynamic model of an unmanned bicycle has been considered. According to this mathematical model, the neuro-fuzzy and PD controllers have been designed to stabilize the bicycle in its straight running motion and roll angle tracking of a sinusoidal path.

In comparison of PD controller, the Neuro-Fuzzy controller had better performance and robustness and it followed a desired roll angle much better than PD.

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