A Heuristic Method for Transmission Network Expansion Planning under Security Constraints

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***Abstract*-- The transmission expansion planning problem in modern power systems is a large-scale, mixed-integer, non-linear and non-convex problem. This paper presents a novel mathematical model to solve security constraints transmission system expansion planning problem via a constructive heuristic algorithm (CHA). The basic idea comes from Garver's work applied to the transportation model; nevertheless the proposed algorithm works with a hybrid model considering security constraints. The hybrid model is a linear one. Proposed CHA finds an acceptable solution in an iterative process, where in each step a circuit is chosen using a sensitivity index and added to the system. Results of two sample test systems shows the effectiveness of the proposed method comparing with the ones in the literature.**

Index term--transmission network expansion planning, constructive heuristic algorithm, security constraints, Hybrid model.

I. INTRODUCTION

In order to make the power system operation viable for a predefined horizon of planning considering minimum cost, the objective of the transmission expansion planning (TEP) problem is to determine *where*, *how many*, and *when* new devices must be added to a network [1]. As TEP problems are non-linear, mixed integer, non-convex optimization problem, and on other hand considered as very complex and computationally demanding, then various optimization techniques have been proposed to solve these problems[2,3]. The most planning researches don't consider the security constraints. In other words, in the literature , the optimal transmission expansion plan is determined without considering the contingencies caused by the outages transmission-line. Thus, in majority of expansion planning proposals that consider security constraints, the planning process is taken into consideration in two phases. In phase 1 a planning process without security is released, then in phase 2 using the expansion plan obtained in phase 1, new circuits are added considering the security criterion. Generally the second phase uses the same strategy for phase 1. The most important advantage of this type of strategy is to get a reliable expansion. On the other hand these methods cannot obtain the optimal expansion plan. Moreover, the expansion plan of phase 1 has the highest influence in the whole expansion plan, and this can be more critical in big and complex systems.

The recent methodologies for solving the transmission planning problem can be divided into three groups: (1) classic optimization algorithms, e.g., Benders decomposition [4] and branch-and-bound approaches [5] (2) Heuristic algorithms [6], (3) meta-heuristics such as simulated annealing (SA) [7], genetic algorithms (GA) [2], tabu search (TS) [8], GRASP [9], etc. In the mathematical point of view, the TNEP is a mixed integer programming and its nondeterministic polynomial yield to the complexity of its algorithm. In general, the mathematics based methods that applied to medium or large scale power systems are time consuming and if the additional constraints, e.g. N-1 security, are taken into account, the computational burden will be more severe. On the other hand, the heuristic based methods normally can obtain a solution with less computational effort but maybe trap in local solution. However, according to the nature of the problem and with an elaborated modification of the search procedure, it is possible to obtain an acceptable result. The meta-heuristic methods, e.g. genetic algorithm, simulate annealing etc., are mostly similar to the heuristic methods. The highlight of meta-heuristic based methods in the search process is an embedded mechanism to escape from the local optima. Therefore, these methods normally are more time consuming than the heuristics ones.

Taking into account the planning period, the planning problem can be divided as a one stage problem that is called the static planning or a multistage transmission network expansion planning problem where the planning horizon can be separated into several stages. In this paper, only the static planning problem is considered.

In this paper, a novel mathematical model to solve security constraints transmission system expansion planning problem via a constructive heuristic algorithm (CHA) is proposed. A constructive heuristic algorithm can finds an acceptable solution in an iterative process. In each step a circuit is chosen

This work was supported by FAPESP under processes 2010/19032-1 and 2011/13995-5.

using a sensitivity index and added to the system. The stopping criterion is to get a feasible solution; where there is no need to add more circuit to the system. The robustness and fastness are the main features of a CHA.

The proposed CHA algorithm uses a hybrid model considering the (N -1) security criterion, which is the most reported criterion in recent researches on transmission network planning. The (N -1) security criterion implies that the system should be expanded in such a way that, if the system gets a line outage, can operate accurately.

II. PROBLEM FORMULATION

A. DC Model

Usually, long-term TNEP as a mixed integer non-linear problem (MINLP) is modeled by a mathematical formulation, which is so-called the DC model. The DC model for TNEP without security constraints is formulated as follows:

$$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} + \alpha \sum_s r_s \quad (1\text{-}1)$$

st.

$$Sf + g + r = d \quad (1\text{-}2)$$

$$f_{ij} - \gamma_{ij}(n_{ij}^0 + n_{ij})(\theta_i - \theta_j) = 0 \quad (1\text{-}3)$$

$$|f_{ij}| \le (n_{ij}^0 + n_{ij})\bar{f}_{ij} \quad (1\text{-}4)$$

$$0 \le n_{ij} \le \bar{n}_{ij} \quad (1\text{-}5)$$

$$0 \le g \le \bar{g} \quad (1\text{-}6)$$

$$0 \le r \le d \quad (1\text{-}7)$$

n_{ij} integer, θ_i unbounded $(i,j)\epsilon\Omega$

Where:

v : investment costs for a predefined horizon.

c_{ij} : cost of a candidate circuit added to the right-of-way $i-j$.

n_{ij} : number of circuits added to the right-of-way $\mathrm{i}-\mathrm{j}$.

n_{ij}^0 : number of circuits in the initial topology.

$\bar{n}_{ij}$: maximum number of circuits that can be added in right-of-way $\mathrm{i}-\mathrm{j}$.

γ_{ij} : susceptance of line $\mathrm{i}-\mathrm{j}$.

θ_i : phase angle at bus i.

f_{ij} : Active power flow through line $\mathrm{i}-\mathrm{j}$.

$\bar{f}_{ij}$: maximum active power flow limit of line $\mathrm{i}-\mathrm{j}$.

r : vector with artificial generators added in each load bus.

α : dummy generation penalty factor.

S : branch-node incidence matrix.

f : a vector with elements f_{ij}.

g: a vector with elements g_k (generation at bus k) whose maximum value is $\bar{g}$.

Ω : Set of all right-of-ways.

Objective function of the DC model containing investment costs of the newly added transmission lines as well as the penalty load curtailment has been shown in Eq. (1-1). In Eq. (1-2), Kirchhoff's Current Law (KCL) in the equivalent DC network is modeled. Eq. (1-3) is an expression of Ohm's law for the equivalent DC network, while Kirchhoff's Voltage Law (KVL) is implicitly taken into consideration. Equations (1-4), (1-5), (1-6) and (1-7) are based on power flow, generator capacity, line numbers' limitations and load shedding vector respectively.

B. Hybrid Model without Security Constraints

A hybrid linear model is obtained assuming that Eq. (1-3) of the DC model (1), i.e. the KVL is satisfied only by the existing circuits [10].

It can be stated by model (2) as follows.

$$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} + \alpha \sum_s r_s \quad (2)$$

S.t.

$$Sf + S^0 f^0 + g + r = d$$

$$f_{ij}^{01} - \gamma_{ij}(n_{ij}^0)(\theta_i - \theta_j) = 0 \;\; \forall\,(i,j)\in\Omega_1$$

$$|f_{ij}^{01}| \le (n_{ij}^0)\bar{f}_{ij} \;\; \forall\,(i,j)\in\Omega_1$$

$$|f_{ij}| \le n_{ij}\bar{f}_{ij} \;\; (i,j)\in\Omega$$

$$0 \le n_{ij} \le \bar{n}_{ij}$$

$$0 \le g \le \bar{g}$$

$$0 \le r \le d$$

θ_i unbounded

Where:

n_{ij}^0*:* circuits of base topology.

Ω_1: set of all added circuits during the iterative process and all prime circuits of base case.

S^{01}: transpose incidence branch-node matrix of the base topology and added topology in previous iterations of algorithm.

f_{ij}^{01}: power flow on path *(i,j)*$\epsilon\Omega$.

C. Hybrid Model with Security Constraints

The proposed mathematical model for the transmission network expansion planning problem with security constraints presents as belows.

$$min\, v = \sum_{(i,j)\in\Omega} c_{ij} n_{ij} \quad (3)$$

S.t.

$$S^p F^p + S^{01^p} F^{01^p} + g^p = d \;\; \forall p \in \mathrm{SC} \quad (3\text{-}1)$$

$$f_{ij}^{01^p} - \gamma_{ij}(n_{ij}^0 + n_{ij}^1)(\theta_i{}^p - \theta_j{}^p) = 0 \quad (3\text{-}2)$$

$\forall\,(i,j)\in\Omega_0, \forall\,(i,j)\in 1,2\,\ldots.nl$ and $(i,j)\ne p,\;\forall p\in\mathrm{SC}$

$$f_{ij}^{01^p} - \gamma_{ij}(n_{ij}^0 + n_{ij}^1 - 1)(\theta_i{}^p - \theta_j{}^p) = 0 \quad (3\text{-}3)$$

$\forall\,(i,j)\in\Omega_0, \forall\,(i,j) = p,\;\forall p\in\mathrm{SC}$

$$|f_{ij}^{01}| \le (n_{ij}^0 + n_{ij}^1)\bar{f}_{ij} \quad (3\text{-}4)$$

$\forall\,(i,j)\in\Omega_0, \forall\,(i,j)\in 1,2\,\ldots.nl$ and $(i,j)\ne p,\;\forall p\in\mathrm{SC}$

$$|f_{ij}^{01}| \le (n_{ij}^0 + n_{ij}^1 - 1)\bar{f}_{ij} \quad (3\text{-}5)$$

$\forall\,(i,j)\in\Omega_0, \forall\,(i,j) = p,\;\forall p\in\mathrm{SC}$

$$|f_{ij}| \le n_{ij}\bar{f}_{ij} \;\; \forall(i,j)\in\Omega \quad (3\text{-}6)$$

$$0 \le n_{ij} \le \bar{n}_{ij} - n_{ij}^1 \;\; \forall(i,j)\in\Omega \quad (3\text{-}7)$$

$$0 \le g^p \le \bar{g} \quad p\in\mathrm{SC} \quad (3\text{-}8)$$

θ_i unbounded and n_{ij} integer

In order to consider the security constraints in TEP, it is necessary to provide a list of congested transmission lines. One strategy is to identify those lines with most frequent outages from the historical data, or based on the system operator experiences. Another strategy is to solve the proposed model without considering security constraints, and identifying the lines based on optimal solution via an active power flows over a percentage of their maximum capacity usually between 80% to 95%, where the main objective is to identify the most overloaded lines in the system. These lines construct the contingency list to consider the N-1 security. In this model SC represents list of congested transmission lines.

III. APPLICATION OF CHA TO TEP

In this section some fundamental components and main characteristic of CHA are presented. In fact, CHA may find a good quality solution in an iterative process. The fastness and robustness are the main characteristics of CHA. In order to obtain a feasible and high quality solution, in each iteration a circuit is added to the network, where the aforementioned circuit is selected based on a sensitivity index. In order to achieve an optimum expansion plan, the load shedding is not acceptable.

General CHA process is explained through different steps as follows:

Step 1: Assuming a base topology as the current topology.

Step 2: Choose a mathematical model for TEP.

Step 3: Solve LP/ NLP to determine parameters used in the sensitivity index defined in CHA algorithm that considers operational conditions. If LP or NLP solution indicates that the system is adequately operate in new additions, it means that a feasible solution is in hand. Then go to step 5.

Step 4: Use a sensitivity index to identify the most attractive circuit. Update the current topology with the chosen circuit, then go to step 3.

Step 5: Sort the added circuits' costs in decreasing order. Using an LP, it verifies that whether the removal of a circuit keeps the system in an adequate operational condition or not. If yes, remove the circuit, otherwise keep it. Repeat circuit removal process until all the circuits have been tested. All added circuits that weren't removed, represent CHA's final solution.

Many CHAs in literature are of two following categorizes.

i) Algorithms that use electrical system performance to make sensitivity index

ii) Algorithms that use the relaxed version mathematical model.

Algorithms similar to least-load-shedding [11] and least effort [12] belong to group *(i)*, and Garver's [13], Villasana–Garver–Salon (VGS) [10] and the algorithm proposed in this paper also belong to group *(ii)*.

In least-load shedding algorithm, sensitivity index tries to identify the circuit that would provide the most significant reduction in load shedding. For this case, in step 3 CHA solves an LP while operation constraint is load shedding. It can be mentioned that sensitivity index is an approximation due to the fact that the selected circuit may not guarantee the least load shedding. Although the selected circuit may provide a reduction in load shedding, it may not facilitate the optimal topology. All these problems may partially arise when the sensitivity index considers the circuits' costs. A major advantage in using either the least-effort algorithm or the least-load-shedding algorithm is that both of them employ DC model directly. The model that is used in Garver's algorithm is transportation model (TM). The TM is a relaxed version of DC model provided from elimination of third constraint in DC model. In fact, TM is a mixed-integer linear optimization problem. Garver's algorithm relaxes the integrality of the investment variable and solves TM, i.e. making $n_{ij} \geq 0$ and solving the problem as an LP. The LP solution might not be feasible for TEP problem, therefore this solution is deployed as a sensitivity index for CHA. The sensitivity index can be defined by Eq. (4).

$$IS = max\{IS_{ij} = n_{ij}\bar{f}_{ij} \ ; \ n_{ij} \neq 0\} \quad (4)$$

Where: n_{ij} is the solution of LP after relaxing integrality of n_{ij}. In Garver's algorithm at each step an LP with the current topology is solved while the number of new circuits might not be an integer that may facilitate a minimum investment. In this regard, Garver algorithms will face with two crucial difficulties. On the other hand, VGS algorithm can find a good solution for DC model than the best CHA ever proposed in the literature. By relaxing the third constraint in DC model, a hybrid model will be produced where an LP solver can be applied to solve such a hybrid model identifying the most important circuit at each step of algorithm. It has worth to be mentioned that in hybrid model, the relaxed constraint will only be considered to those circuits of the current topology.

IV. CHA FOR EXTENDED HYBRID MODEL

Unlike the CHAs in literature that solves only a simple model without considering security constraints, the proposed CHA, works with an extended hybrid model that considers security constraints. The CHA solves model(3) after relaxing the integrality of investment variables, i.e. the integer n_{ij} is changed to $n_{ij} \geq 0$. Another feature presented in this algorithm is that every circuit added in the process must comply with both KCL and KVL which means compatibility between current solution and the DC model solution. The major drawback of this method is that at each CHA step a very large LP must be solved where it gets considerable for large scale power systems. The proposed CHA employed in this work is as follows:

Step 1: Assume the base topology as current topology.

Step 2: Solve LP to determine those parameters used in the sensitivity index Eq. (4). If LP solution indicates that the system is adequately operate in new additions, it means that a new solution for DC model has been obtained, then go to step 4.

Step 3: Use sensitivity index Eq. (4) to identify the most attractive circuit. Update the current topology with the selected circuit, then go to step 2.

Step 4: Sort the added circuits in a descending order of costs. Using an LP, it verifies that whether the removal circuit keeps the system in adequate operational conditions or not. If yes, remove the circuit, otherwise keep it. Continue circuit removal until all circuits have been examined. All added circuits that weren't removed represent the CHA's solution.

It can be notified that although this CHA uses a hybrid linear model to identify the best circuit to add in an iterative process, it complies with both Kirchhoff's Laws after adding a new circuit, thus the final solution is also a feasible solution of the DC model (1).

V. TESTS AND RESULTS

The proposed algorithm to solve the transmission expansion planning problem with security constraints has tested using two mostly used electrical power systems in the specialized literature. The first system is the system that originally proposed by Garver and the second is the IEEE 24-bus system.

The comparison of results is presented for IEEE 24 bus system with the one obtained with basic binary GA, bacterial foraging-differential evaluation algorithm (BF-DEA) and harmony search algorithm (IHA) to show the effectiveness of the proposed algorithm. The algorithm has implemented using an AMPL structure using CPLEX solver to solve the LP problem at each step of the constructive algorithm.

A. Garver 6-bus test system

The Garver system has 6 buses and 15 candidate branches. The total demand is 760 MW and maximum possible number of added lines per branch equals five. The electrical system data for this system has extracted from[14]. The initial topology is shown in fig. 1.

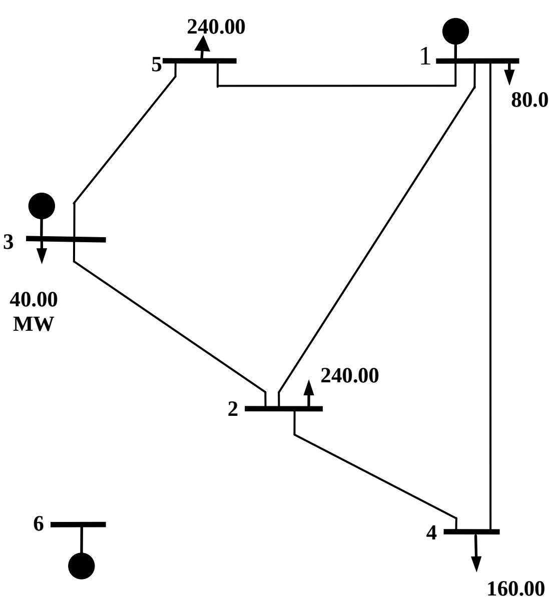

fig. 1. Garver System

1) Plan without security constraints

The optimal solution of the expansion planning problem without security constraints is equal to *v* = US$ 200,000 and the following lines are added: $n_{2-6}=4$, $n_{3-5}=1$ and $n_{4-6}=2$. The CHA converges after solving 11 LPs without removing any circuit in step 4.

The sequence of adding line is as follows:
n_{4-6}, n_{4-6}, n_{2-6}, n_{2-6}, n_{2-6}, n_{3-5} and n_{2-6}.

2) Plan with security constraints

The planning with security constraints to this system can be found using the proposed CHA, resulting an investment of *v = US$ 300,000* considering the following added lines: $n_{2-6}=5$, $n_{3-5}=2$, $n_{4-6}=3$ and $n_{2-3}=1$.

The proposed CHA converges after solving 16 LPs and without removing any circuit in step 4. The sequence of adding line is as follows: n_{2-6}, n_{4-6}, n_{2-6}, n_{2-6}, n_{2-6}, n_{3-5}, n_{4-6}, n_{3-5}, n_{4-6}, n_{2-6} and n_{2-3}.

Table I shows the results of each iteration, *Vlp* is the investment provided by LP, *V* is the partial investment during the iterative process and *Sln* shows the selected circuit in each iteration.

B. IEEE 24-bus system

This system consists of 24 buses, 41 candidate circuits and 8550MW of total demand which is shown in Fig. 2. Maximum possible number of added lines per branch equals 3. The electrical data and generation/load data have been taken for plans' G1–G4 of [16].

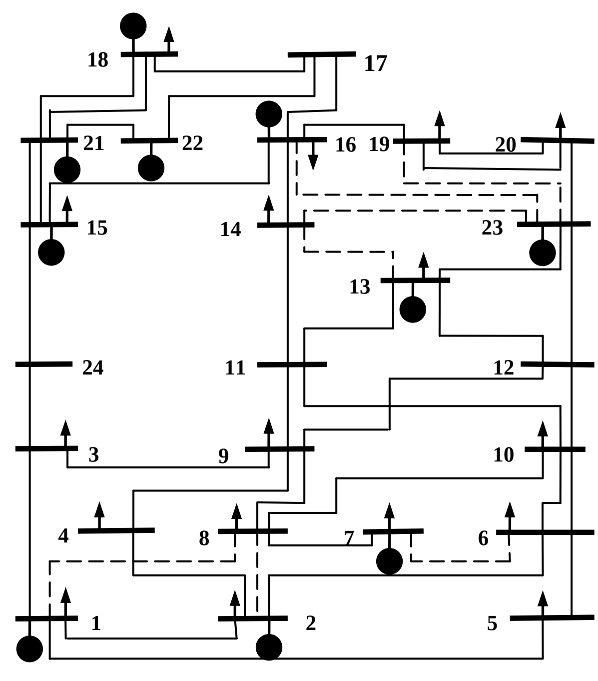

fig. 2 IEEE 24-bus System

TABLE I results for Garver system iteration-by-iteration

Iteration →	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1-2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1-3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1-4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1-5	0.2960	0.3513	0.3130	0.1798	0.1841	0.1849	0.0000	0.0722	0.0744	0.0733	0.0729	0.0000
1-6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2-3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0778	0.0756	0.0767	0.0771	0.0000
2-4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2-5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2-6	3.1028	2.6037	2.5870	2.5005	1.4996	0.4871	0.3750	0.2079	0.2393	0.2390	0.0000	0.0000
3-4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3-5	1.1040	1.0487	1.0870	1.2202	1.2159	1.2151	0.8000	0.7620	0.0000	0.0000	0.0000	0.0000
3-6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1500	0.0000	0.0000	0.0000	0.0000	0.0000
4-5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4-6	2.3472	2.8463	1.8630	0.9495	0.9504	0.9629	0.9250	0.7392	0.7362	0.0000	0.0000	0.0000
5-6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SLn	n_{2-6}	n_{4-6}	n_{2-6}	n_{2-6}	n_{2-6}	n_{3-5}	n_{4-6}	n_{3-5}	n_{4-6}	n_{2-6}	n_{2-3}	----
Vlp	191.5	191.5	161.5	131.5	101.5	71.5	62.2	46.652	32.264	10.170	3	0
V	30	60	90	120	150	170	200	220	250	280	300	300

1) Plan without security constraints

Considering generation plan G1, the CHA founds the expansion plan of v = US$ 438,000,000 with the following topology:

$n_{5-1}=1$, $n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{14-16}=1$, $n_{15-21}=1$, $n_{15-24}=1$, $n_{16-17}=2$, $n_{16-19}=1$, $n_{17-18}=1$.

Taking into account the plan G2, the expansion plan of v = US$ 494,000,000 is derived from the proposed CHA founds considering the following topology:

$n_{5-1}=1$, $n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=1$, $n_{10-11}=1$, $n_{14-16}=2$, $n_{15-21}=1$, $n_{15-24}=1$, $n_{16-17}=2$ and $n_{17-18}=1$.

For generation plan G3, the CHA founds the expansion plan of v = US$ 218,000,000 with the following topology:

$n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{14-16}=1$, $n_{16-17}=1$ and $n_{20-23}=1$.

And finally considering generation plan G4, the expansion plan of v = US$ 376,000,000 using the proposed CHA with the following topology is in hand:

$n_{3-24}=1$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{12-13}=1$, $n_{14-16}=1$, $n_{15-24}=1$ and $n_{17-18}=1$.

1) plan with security constraints

The investment of the expansion planning considering security constraints for plan G1 using the proposed CHA is v = US$ 949,000,000 where the added lines are as follows:

$n_{1-5}=1$, $n_{2-8}=1$, $n_{3-24}=2$, $n_{4-9}=1$, $n_{5-10}=1$, $n_{6-7}=2$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-11}=1$, $n_{11-14}=1$, $n_{14-16}=2$, $n_{15-16}=1$, $n_{15-24}=2$, $n_{16-17}=3$, $n_{16-19}=2$ and $n_{17-18}=2$.

The CHA converges after solving 42 LPs and without removing any circuit in step 4. Table 2 shows the iterations' results and sequence of adding lines.

Considering generation plan G2, the CHA founds the expansion plan of v = US$ 964,000,000 with the following topology:

$n_{1-5}=1$, $n_{3-24}=2$, $n_{4-9}=1$, $n_{6-10}=2$, $n_{7-8}=1$, $n_{9-11}=1$, $n_{10-11}=2$, $n_{11-14}=1$, $n_{14-16}=3$, $n_{15-21}=1$, $n_{15-24}=2$, $n_{16-17}=3$ $n_{17-18}=2$ and $n_{2-8}=1$.

For this plan the CHA converges after solving 124 LPs by removing circuits of n_{6-7}, n_{6-7}, n_{2-8}, n_{6-7} and n_{15-16} in step 4 respectively.

Considering generation plan G3, the investment of expansion plan founded by CHA is v = US$ 722,000,000 with the following topology:

$n_{1-5}=1$, $n_{2-8}=1$, $n_{3-24}=1$, $n_{4-9}=1$, $n_{6-7}=2$, $n_{6-10}=1$, $n_{7-8}=2$, $n_{10-12}=1$, $n_{12-13}=1$, $n_{14-16}=2$, $n_{15-16}=1$, $n_{15-24}=1$, $n_{16-17}=2$, $n_{17-18}=1$ and $n_{20-23}=1$.

For this plan the CHA converges after solving 35 LPs and without removing any circuit in step 4.

Finally for plan G4, the CHA found the expansion plan of v = US$ 818,000,000 with the following topology:

$n_{1-5}=1$, $n_{2-8}=1$, $n_{3-24}=2$, $n_{4-9}=1$, $n_{6-7}=2$, $n_{6-10}=2$, $n_{7-8}=1$, $n_{9-11}=1$, $n_{10-12}=2$, $n_{12-13}=1$, $n_{14-16}=2$, $n_{15-24}=1$, $n_{16-17}=2$ and $n_{17-18}=1$.

For this plan the CHA converges after solving 35 LPs and without removing any circuit in step 4.

Table II contains a comparison between the results of the proposed method and the methods in the literature. With considering cost, it shows that the proposed method can get better results than the other presented methods. The superiority of the method is manifested when the number of fitness function or LP is taken into consideration, the reported solutions in literature solves thousands fitness functions while the proposed method can get results after solving a negligible number of LP. It must be mentioned that the reported meta-heuristic methods have to solve a number of LP as fitness function then here the comparison criterion is the number of LP.

TABLE II Comparison of cost and number of fitness function

Plan		G1	G2	G3	G4
Cost [10^6 US$]	**IHA[3]**	964	942	837	882
	GA[3]	978	977	903	899
	BFDEA[3]	975	974	898	882
	CHA	949	964	722	818
Number fitness function (LP)	**IHA[3]**	118280	20450	58400	220500
	GA[3]	1945090	313167	2753166	2690833
	BFDEA[3]	1157900	737300	553500	2753166
	CHA	42	124	35	35

VI. CONCLUSIONS

A mathematical model and a constructive heuristic algorithm to solve the transmission network expansion planning problem considering security constraints have been presented. The results obtained using small- and medium-size commonly used systems show the good performance of the proposed methodology. A comparative analysis between the results obtained using the proposed CHA and the results obtained using basic binary GA, bacterial foraging-differential evaluation algorithm (BF-DEA) and harmony search algorithm (IHA) for IEEE 24 bus systems shows the effectiveness and superiority of the proposed approach. On the other hand the proposed CHA provides a better (low cost) solution for all the cases while the less number of fitness function evaluations is needed.

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