Geo-Morphology Modeling in SAR Imagery Using Random Fractal Geometry

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***Abstract*— Geological formations has different behaviors against weathering and erosion and it causes difference in geo-morphology. Geological mapping needs the ability of lithological discrimination on the basis of geo-morphology, and this capability is not fully accessible via optical remote sensing. Since radar spectral windows in electromagnetic spectrum is independent of solar energy and can penetrate clouds and particularly sensitive to surface parameters, they are considered to be useful for studies of the surface geological morphology. In order to discriminate the surface geometric pattern and differentiate top-geological formations surface, it is required to model the softness and roughness of surface according to the radar signal backscattering. Fractal geometry is much more capable to describe natural phenomena than conventional geometry. Fractal geometry has been used several times in literature in order to improve the radar backscattering models. This paper compares application of different autocorrelation functions for the most famous model in this manner, integral equation model (IEM) benefiting random fractal geometry. Trying to improve geological mapping of Dehloran geological formation (western boundary of Ilam in IRAN), the results display the level of effectiveness of the conventional autocorrelation function.**

Keywords—geological formations, SAR images, roughness modeling, backscattering coefficient

I. INTRODUCTION

Detection of top-geological structures cannot be possible via optical imagery especially in large regions; since study of geological morphology to some extents is not possible by passive remote sensing. Because of independence of microwave sensors to climate changes, and especially their sensitivity to surface parameters, SAR technology is suitable for geomorphology and earth surface studies.

In Dehloran geological formation, some geological structures containing lithologies like Marne, are more affected by alteration and weathering and consequently are physically smooth. In contrary, there are some other structures which are less affected by physical and chemical erosion, and have rough and rigid face, such as Anhydride lithology. In the process of mapping this region on geological maps, discrimination among the different top-geological structures cannot be possible via available optical imagery; since geological morphologies to some extents are not differentiable by passive remote sensing. Geological morphology modeling by SAR data needs to have topography and micro-topography model of the surface.

Geological morphology modeling by SAR data needs to have topography and micro-topography model of the surface. Roughness parameters are highly dependent to measurement scale which is the SAR signal wavelength in this study. Natural phenomena cannot be qualitatively modeled via conventional geometry; in contrast, Random Fractals Geometry is much more powerful in modeling natural shapes [1].

In this paper different autocorrelation functions for the most famous model in this manner, integral equation model (IEM) is applied and by using fractal autocorrelation function, instead of using the Gaussian and exponential functions [1], we try to improve geological mapping of morphology. In other words, this paper tries to improve precision of parameters estimation in Integral Equation Model (IEM) [2], and then by considering geomorphology, to increase quality and precision of geological maps. Verification of modeling processes are applied to ALOS SAR data of Dehloran geological structure to improve geological mapping precision.

II. INTEGRAL EQUATION MODEL (IEM) AND ROUGHNESS PARAMETERS

Standard theoretical models of backscattering, are: Geometric Optics Model (GOM) and Physical Optics Model (POM) and Small Perturbation Model (SPM). Geometric Optics Model, for very rough surfaces, Physical Optics Model, for medium roughness and Small Perturbation Model, for very smooth surfaces are used. Fung and Chen have developed Integral Equation Model (IEM) as a physically based electro-magnetic transfer model IEM via combination of the GOM, POM and SPM, and constructed a more applicable model which can tolerate a really wide range of roughness dimensions, theoretically, IEM is not restricted to any special situation [1]. As defined, IEM relates backscattering coefficients to roughness parameters of the surface, dielectric permittivity and magnetic permeability, and the local incidence angle. The co-polarized backscattering coefficient has been explained as [3]:

$$\sigma_{pp}^{0} = \frac{k^2}{4\pi} e^{-2k^2\sigma^2 cos^2\theta} \sum_{n=1}^{+\infty} \left| I_{pp}^{n} \right|^2 \frac{W^{(n)}(2ksin\theta, 0)}{n!} \qquad (1)$$

where

$$I_{pp}^{n} = (2k\ \sigma\ cos) f_{pp} \exp(-k^2\sigma^2 cos^2\theta) + (k\ \sigma\ cos)^n F_{pp} \qquad (2)$$

and pp, polarization (hh or vv); k, wave number ($k = \frac{2\pi}{\lambda}$: λ is the wavelength), θ is the local incidence angle, σ, the surface rms-height, $W^{(n)}$, fourier transform of n^{th} power of the correlation function, and f_{hh}, f_{vv}, F_{hh} and F_{vv} are approximated by:

$$f_{hh} = \frac{-2R_h}{cos\theta}$$

$$f_{hh} = \frac{2R_v}{cos\theta}$$

$$F_{hh} = 2\frac{sin^2\theta}{cos\theta}\left[4R_h - \left(1 - \frac{1}{\varepsilon}\right)(1 + R_h)^2\right]$$

$$F_{vv} = 2\frac{sin^2\theta}{cos\theta}\left[\left(1 - \frac{\varepsilon\, cos^2\theta}{\varepsilon - sin^2\theta}\right)(1 - R_v)^2 - \left(1 - \frac{1}{\varepsilon}\right)(1 + R_v)^2\right]$$

The horizontally and vertically polarized Fresnel reflection coefficients, R_h and R_v, are described as:

$$R_h = \frac{cos\theta - \sqrt{\varepsilon - sin^2\theta}}{cos\theta + \sqrt{\varepsilon - sin^2\theta}}$$

$$R_v = \frac{\varepsilon\, cos\theta - \sqrt{\varepsilon - sin^2\theta}}{\varepsilon\, cos\theta + \sqrt{\varepsilon - sin^2\theta}}$$

ε is the dielectric constant. Considering $C(\rho)$, the surface autocorrelation function (ACF), and J_0 as zeroth order Bessel function, surface power spectrum, $W^{(n)}$ in IEM can be defined as:

$$W^{(n)}(K) = \int_{\rho=0}^{\rho=+\infty} C(\rho).\rho.J_0(K_\rho)d\rho \qquad (3)$$

Gaussian and exponential functions are two special cases of ACF, which will be described in the next section.

Calculation of the model parameters from SAR signal backscattering coefficient is not directly possible, because of the model complexity and some other strategies must be pursued. In this paper Look Up Table (LUT) method is employed; so for this purpose a table of different possible values of roughness parameters/dielectric constant and corresponding backscattering coefficients tabulated.

In sections *A*, *B* and *C*, respectively, the three main specifications for the radar Backscattering study, the rms-height, correlation length and autocorrelation function are defined.

A. Heights Root Mean Square (rms-height)

The root mean square of surface heights (rms-height) defines the variation in surface elevation above an arbitrary plane and is used to be calculated on the basis of a one-dimensional discrete surface profile measurement consisting N points with elevations z_i [4]:

$$s = \sqrt{\frac{1}{N}\left[\left(\sum_{i=1}^{N} z_i^2\right) - N\bar{z}^2\right]} \qquad (4)$$

where

$$\bar{z} = \frac{1}{N}\sum_{i=1}^{N} z_i \qquad (5)$$

In (4), s represents standard deviation of the surface micro-topography discrete heights.

B. Correlation Length

The level of surface heights uniformity over a finite profile of the surface is usually described by Correlation Length [4]. In other words, the horizontal variations of the surface heights is called correlation length. Unlike the simplicity of this definition, measurements of the correlation length is complicated. The calculated values for this parameter via different ways are extremely variable and also greatly depends on the length of the sampling profile length [6]. As a typical methodology, Davidson et al. (2003) has proposed a linear interpolation on the correlation function of the heights:

$$l = \left(e^{-1} - C(\rho_1)\right)\frac{\rho_2 - \rho_1}{C(\rho_2) - C(\rho_1)} + \rho_1 \qquad (6)$$

where $C(\rho)$ is the autocorrelation function, ρ_1 and ρ_2 are two arbitrary points. On the basis of the parameter definition, in this equation, it is considered that $C(l) = e^{-1}$.

C. Autocorrelation Function

The normalized autocorrelation function, for $\rho = j\Delta x$, where Δx is the spatial resolution of the profile, is given by:

$$C(\rho) = \frac{\sum_{i=1}^{N-j} z_i z_{i+j}}{\sum_{i=1}^{N} z_i^2} \qquad (7)$$

In order to fully characterize the ACF of a surface, a discretization interval, used to sample the profile, should be at least as small as one tenth of the correlation length.

In backscattering models, often two types of ACFs, the exponential and the Gaussian autocorrelation functions are being used. The exponential ACF is given by:

$$C(\rho) = e^{-|\rho|/l} \qquad (8)$$

and the Gaussian function;

$$C(\rho) = e^{-\rho^2/l^2} \qquad (9)$$

where *l*, is the correlation length.

III. IEM MODELING METHOD USING FRACTAL GEOMETRY

The most famous improvement for backscattering modeling using fractal geometry is using fractal ACF instead of Gaussian or exponential one.

Eqs. (8) and (9), show the exponential and Gaussian ACFs, respectively. Likewise fractal correlation function is [1]:

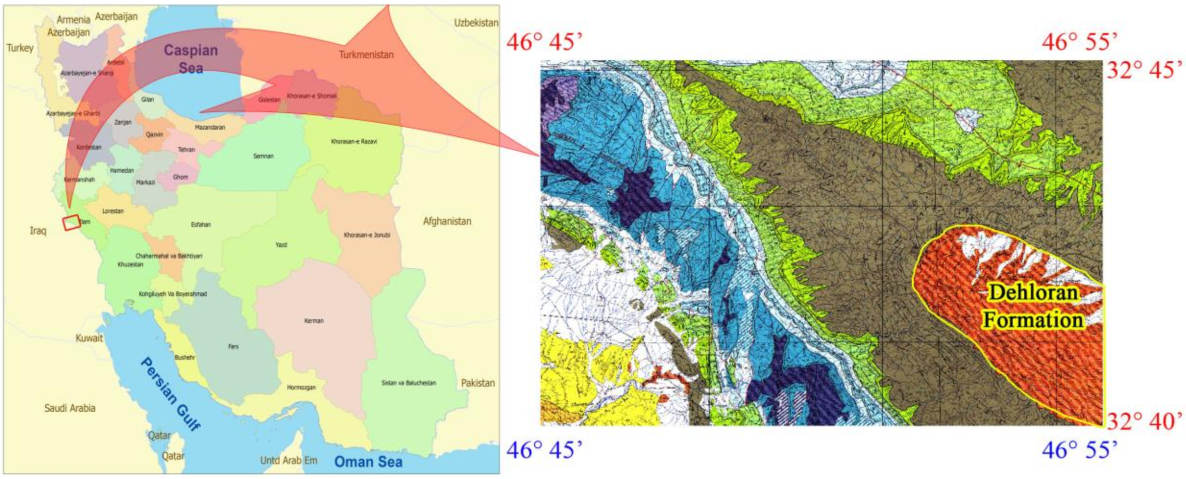

Fig. 1 Case Study region in western part of Dehloran geological formation

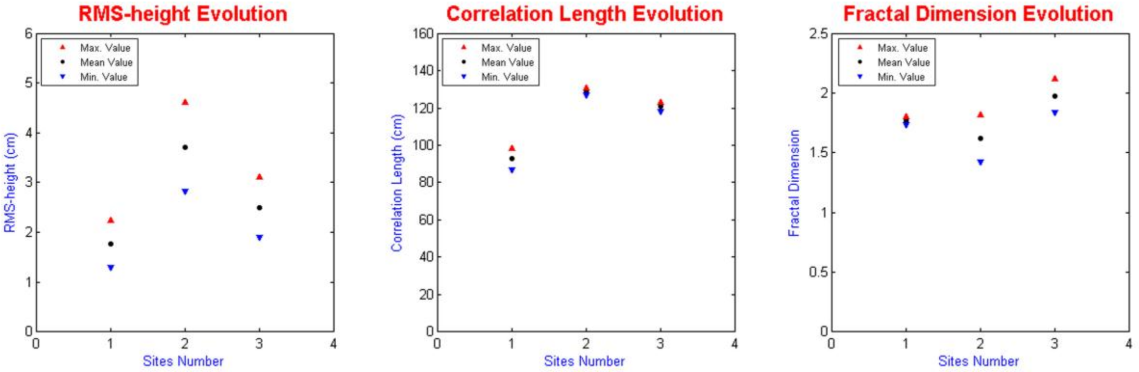

Fig. 2 Case Studies’ surface roughness parameters, rms-height, Correlation Length, Fractal Dimension

$$C(\rho) = e^{-\rho^{\tau}/l^{\tau}} \qquad (10)$$

where, on the basis of experimental data, parameter τ has a linear relation with fractal dimension and the relation is computable via: $\tau = -1.67\ D + 3.67$, in which, D is the fractal dimension of the earth surface and can be calculated via various methods. In natural surfaces which have more complexity, fractal functions are more appropriate than the exponential and Gaussian correlation functions. Since the exponential and Gaussian functions are particular forms for particular situations of fractal function [1].

IV. IMPLEMENTATION, RESULTS AND DISCUSSION

A. Case Study

As the application case study, western part of Dehloran geological formation is selected which is located in the following coordinates:
Longitude: *46°45'* to *46°55'*
Latitude: *32°40'* to *32°45'*

Fig. 1 illustrates geographic and geological position of the case study.Geomorphology of the region, which is depicted in the image of Fig. 1 is a geological section of Dehloran structure and obviously different members of Pabdeh, Asmari and Kalhor formations are figured out. Different decay properties of these geological units are the reason of different surface morphology. Regional lithologies contain of limestone, dolomite, marl and anhydride. Discrimination of the units for geological mapping and interpretation of optical images needs in situ hardness and softness measurements; and without considerations of the surface morphology is approximately impossible.

Fig. 2 depicts the variation of the three roughness parameters: rms-height, Correlation Length, Fractal Dimension. The rms-height has the most variations among other two parameters. Site 1 has more diversity range among the sites, In contrary, fractal dimension of the sites 2 and 3 are more diverse than the site 1.

Fig. 3 illustrates the three case studies position on the SAR image. Site1 is on a plain region, site2, on a mountainous region and site 3 is on foothill, which can be considered as a transition zone between mountain and plain regions.

B. Implementation and Results

As previously mentioned, the three study sites position are shown in Fig. 3. Surface roughness has been measured based on digital elevation model of the site for a total number of 20 pixels and the dielectric constant has been extracted from the presented tables of [5]. Fig. 4 illustrates backscattering

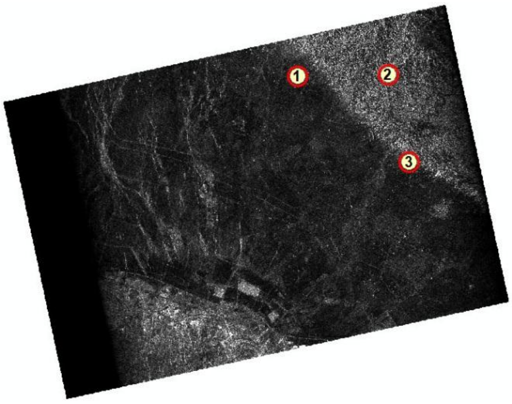

Fig. 3 Case Studies' surface roughness parameters, rms-height, Correlation Length, Fractal Dimension

coefficient calculated by IEM based on conventional geometry (Eq.1) in both *hh* and *vv* polarizations as a function of backscattering coefficient measured from SAR images. Distance of points far from the diagonal line shows the error of the simulation.

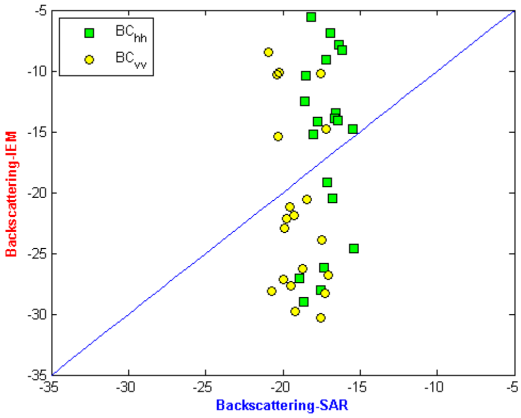

Fig. 4 Simulation accuracy using backscattering equation IEM in two hh and vv polarizations for 20 sample points of the study area, based on conventional geometry. Number and distance of points far from the diagonal line shows the error of the simulation.

As mentioned in the previous section, a method is already presented to utilize the benefits of fractal geometry in backscattering electromagnetic models; which is implemented on the case study data and the comparative results are presented in Figs. 5 and 6. As described for Fig. 4, these graphs show the accuracy level of the IEM simulation.

According to the section A, typically, surface spectrum in IEM equation must be calculated through the Fourier spectrum of the correlation function in the equations (8), (9) and (10). Also, equation (10) as a general function depending on the value of τ, can replace the functions (8) and (9). For the data of study area, the value of $\tau = 1.2$, represents the optimum results comparing to measured backscattering coefficients on SAR image, which is used in the graph of Fig. 6.

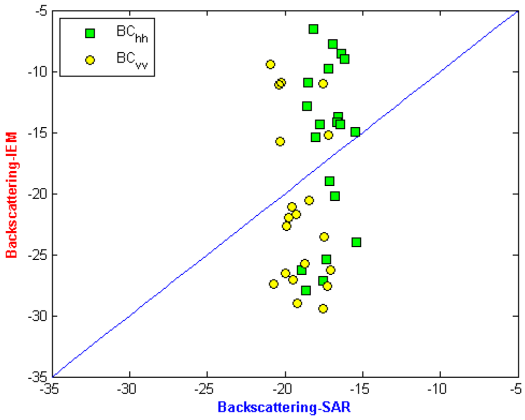

Fig. 5 Simulation accuracy using backscattering equation IEM in two hh and vv polarizations for 20 sample points of the study area, using fractal autocorrelation function ($\tau = 1.2$) instead of Gaussian and exponential functions.

Fig. 6, illustrates the simulation accuracy using IEM backscattering equation in two *hh* and *vv* polarizations for sample points of the study area, using the correlation length calculated via fractal dimension parameter.

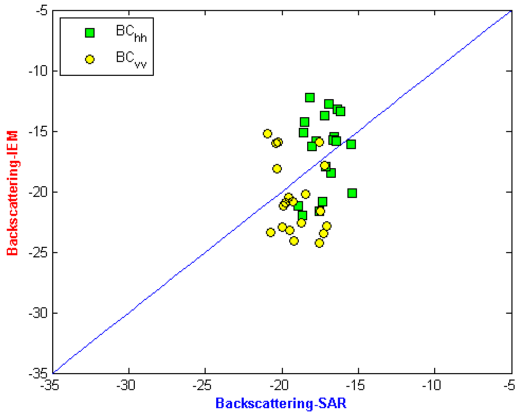

Fig. 6 Simulation accuracy of backscattering equation IEM in two hh and vv polarizations for sample points of the study area, using the correlation length calculated via fractal dimension parameter.

Table 1 presents the statistical analysis of the results acquired through these methods compared to each other.

TABLE I. METHODS RESULTS STANDARD DEVIATION FOR INTEGRAL EQUATION MODEL (IEM)

	Original IEM - Gaussian ACF	Original IEM - Exponential ACF	Original IEMm - fractal ACF ($\tau = 1.2$)
hh-polarization	10.023	7.518	6.891
vv-polarization	9.666	7.249	6.645

V. CONCLUSION

Due to irregular and fractal nature of the surface roughness, electromagnetic backscattering modeling of radar signals using fractal geometry calculates surface parameters closer to actual values. The model IEM is for three types of ACFs and for 20 sample points on three different sites is tested. The graphs and the deviation table, demonstrate obviously the effectiveness of fractal ACF. The studied fractal ACF is implemented with the available linear interpolation which relates fractal dimension and correlation length, more studies on this interpolation can be planned for future studies.

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