Scientific Cooperations International Workshops on Electrical and Computer Engineering Subfields
22-23 August 2014, Koc University, ISTANBUL/TURKEY

Vibration Control of an Elastic Structure using Piezoelectric Sensor and Actuator with Cantilevered Beam as a Case Study

Mohammad Jafari
Graduate Student, Mechanical Engineering and Manufacturing Department
Universiti Putra Malaysia (UPM)
Serdang, Selangor DarulEhsan, Malaysia
al.m.jafari@gmail.com

Harijono Djojodihardjo
Professor and Corresponding Author
Aerospace Engineering Department
Universiti Putra Malaysia (UPM)
Serdang, Selangor DarulEhsan, Malaysia
harijono@djojodihardjo.com

***Abstract*—As a baseline for treating flexible beam attached to central-body space structure, the generic problem of a cantilevered Euler-Bernoulli beam with piezoelectric sensor and actuator attached as appropriate along the beam and its control is solved in great detail. For comparative study, three generic configurations of the combined beam and piezoelectric elements are solved by numerical methods and compared with available analytical and experimental results. The equation of motion of the beam is obtained by using Hamilton's principle, and the baseline problem is solved using finite element method. An in-house computational routine is developed for various applications.**

Keywords*—Euler-Bernoulli Beam Theory, Finite Element Method, Hamiltonian Mechanics, Piezoelectric Material, Active Vibration Control, Structural Dynamics

I. INTRODUCTION

Vibration control of light-weight structures is of great interest of many studies and investigations[1-3]. The high cost of sending heavy masses and large volumes into space has prompted the wide utilization of light-weight structures in space applications, such as antennas, robot's arms, solar panels. A model of such set-up is exemplified in Fig. 1[2]. These kinds of structures are largely flexible, which results in lightly damped vibration, instability and fatigue. To suppress the adverse effect of vibration, sophisticated controller is needed.

Fig. 1. Solar Panel on a typical satellite

Active control approaches are widely reported in the literatures for the vibration control of structures. The active control approach makes use of actuators and sensors to find out some essential variables of the structure and suppress its vibration through minimizing the settling time and the maximum amplitude of the undesirable oscillation. This method requires a specific level of understanding about the dynamic behavior of continuous structures via mathematical modelling[4, 5]. Selecting adequate sensor and actuator is an important issue in active vibration control[6, 7]. The conventional form of sensor and actuator, such as electro-hydraulic or electro-magnetic actuator, are not applicable to implement on the light-weight space structures. Thus, in recent years, a new form of sensor and actuator has been studied using smart materials, such as shape memory alloys and piezoelectric materials. The definition of smart material may be expressed as a material which adapts itself in response to environmental changes. Among smart materials, piezoelectric materials are widely studied in literatures, since they have many advantageous such as adequate accuracy in sensing and actuating, applicable in the wide frequency range of operations, applicable in distributed or discrete manner and available in different size, shape and arrangement.

Space structures can be simplified mostly in the form of beam and plate. In this investigation, only beam theory is considered. From the fundamental beam theory Euler and Bernoulli developed one of the most practical and straightforward theories; however, as beam theory progresses, more sophisticated and accurate theories are developed like Rayleigh and Timoshenko beam theories. Euler-Bernoulli beam theory is applicable to thin and long span, for which plane sections can be assumed to remain plane and perpendicular to the beam axis, and shear stress and rotational inertia of the cross section can be neglected. Solar panel and antenna are very flexible and slender, so that Euler-Bernoulli beam theory can be considered. The equation of motion of the beam may be obtained using Newtonian mechanics, or analytical mechanic approaches such as Hamilton's method and Lagrange method[8, 9]. Hamiltonian mechanics is an elegant and convenient approach, since scalar equation of motion of the beam and boundary conditions are obtained simultaneously. The partial differential equation of motion of the beam can be solved by analytical methods such as separation of variables, or numerical methods such as finite element method. Since these structures are flexible, there is a need for control in order not to disturb the functionality of the space structure as a whole; for example solar panels should be able to do some maneuver to point toward the sun, where vibration can occur in the panel. In order to facilitate maneuvering and attitude vibration control, this study is focused on how the flexibility can be controlled for well-behaved space structural dynamics. There are several ways to control the vibration[10]. Then the effort is aimed for devising a simple and effective controller to manipulate the vibration of a flexible structure. One of the adequate and simple controllers is Proportional-Integral-Derivative (PID) controller, which is classified as classic and linear controller[11]. PID controller minimizes the steady state error of the system. Linear Quadratic Regulator (LQR) controller is another convenient method. LQR is expressed as optimal and modern controller, which is based on minimizing the cost function of a dynamic system[11]. To develop a successful operation, most controllers have been developed for a finite number of natural modes where the controllability and observability conditions are met.

In order to design a system for controlling the vibration of structures, a good knowledge of several particular areas such as dynamics, control theory and dynamics of sensing and actuating transducers are required; however, many assumptions are considered in the literatures to simplify the problem. In this study, the need for the vibration control of beam structure is explored. The dynamic behavior of the beam under transverse vibration is studied using Hamilton's principle. Both analytical and finite element method are utilized to solve the equation of motion. First and second natural modes of the beam are considered for controlling the vibration of the beam, since these modes have more significant effect than other higher modes in the dynamic analysis of the beam. Also, the mode reduction can simplify the problem in control. PID and LQR control are considered to control vibration through PZT(Lead ZirconateTitanate – $PbZr_xTi_{1-x}O_3$) actuator.

In following sections, formulation of problem is represented in section two. The general mathematical model of the Euler-Bernoulli beam and finite element method to solve the equation of motion is discussed in section 3. In section four, modal order reduction and state-space form of the system is described. The control strategies utilized are defined in section five. Finally, the results from three case studies are discussed in section six.

II. Formulation of Generic Problems

Following a series of previous investigation on the analysis of impact resilient structure[12-15], and vibration analysis of an elastic clamped cantilever beam[16], the main aim of this investigation is to design a straightforward and convenient controller for suppressing the transverse vibration in a cantilever aluminum flexible beam through using sensing and actuating transducers. The Euler-Bernoulli beam theory is utilized to model the flexible beam with piezoelectric patches. Three different piezoelectric material configurations on the aluminum beam are considered for comparative study. Finite element method is utilized to achieve the natural frequencies and natural modes. Case study one is validated by analytical solution. Previous experimental result is used for validation of case study two.

To design the controller, two first major natural modes of the beam vibration are considered, since other natural modes has insignificant effect[4, 7, 17].The dynamic equation of the beam is transferred to state space form in order to design controllers. Two controllers are designed for each case study: PID controller and LQR controller with observer. These controllers are easy to perform and effective to suppress the vibration of the beam.

III. Modelling of The beam

The general equation of motion of the beam patched with piezoelectric material can be described as follows, which can be derived by using Hamilton's principle(a detailed elaboration is carried out in a companion paper [18]).

$$\bar{\rho}(x)\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left(\bar{I}(x)\frac{\partial^2 w}{\partial x^2}\right)=\frac{\partial^2 M(x,t)}{\partial x^2} \quad (1)$$

where $\bar{\rho}$ and $\bar{I}$ define as

$$\begin{aligned}\bar{\rho}(x)&=\rho_{bm}A_{bm}+\rho_{sn}A_{sn}K_{sn}(x)+\rho_{ac}A_{ac}K_{ac}(x)\\ \bar{I}(x)&=E_{bm}I_{bm}+E_{sn}I_{sn}K_{sn}(x)+E_{ac}I_{ac}K_{ac}(x)\end{aligned} \quad (2)$$

where subscripts *bm*, *sn* and *ac* represent the original beam, the sensor layer and the actuator layer, respectively. ρ,A, E and I are density, area of the cross section, elasticity modulus and the moment of the inertia, respectively. K(x) represents the sensor or actuator location on the beam through Heaviside function.$M(x,t)$ is actuator moment on the beam, which defines as

$$\begin{aligned}M(x,t)&=bE_{ac}\left(\frac{h_{bm}}{2}\right)d_{31}V(x,t)\\ M(x,t)&=C_{ac}V(x,t)\end{aligned} \quad (3)$$

b and h is the width and thickness of the layer. d31 is piezoelectric strain constants and Cac is a constant that expresses the moment produced per unit control voltage. The voltage generated by sensor can be expressed as

$$\begin{aligned}V_{sn}&=-E_{sn}g_{31}\frac{h_{sn}}{(x_{sn,2}-x_{sn,1})}\left(\frac{h_{bm}+h_{sn}}{2}\right)\left(\frac{\partial w}{\partial x}\bigg|_{x_{sn,1}}^{x_{sn,2}}\right)\\ V_{sn}&=C_{sn}\cdot\left(\frac{\partial w}{\partial x}\bigg|_{x_{sn,1}}^{x_{sn,2}}\right)\end{aligned} \quad (4)$$

where x_{sn}represents the location of beginning and end points of the sensor and g_{31} is the piezoelectric voltage constant. C_{sn}can be defined as the sensor constant.

To obtain the natural frequencies and modes of the system through finite element method, the external moment assumed to be zero.The finite element method due to Galerkin method is utilized to solve (1) for natural frequencies and natural modes[19]. To implement Galerkin method, a test function $\phi(x)$ is multiplied by (1) and integrated with respect to x over the domain.

$$\int_0^l \phi(x)\left\{\bar{I}\frac{d^4W(x)}{dx^4}-\bar{\rho}\omega^2W(x)\right\}dx=0 \quad (4)$$

After integrating by parts and applying the boundary conditions, (5) is obtained.

$$\int_0^l \frac{d^2\phi(x)}{dx^2}\left(\bar{I}\frac{d^2W(x)}{dx^2}\right)dx - \int_0^l \phi(x)\left(\bar{\rho}\omega^2 W(x)\right)dx = 0 \tag{5}$$

which can be rewritten in following form.

$$\sum_{j=1}^{n}\int_{(j-1)h}^{jh}\bar{I}\left[\frac{d^2\phi(x)}{dx^2}\right]\left[\frac{d^2W(x)}{dx^2}\right]dx - \omega^2\sum_{j=1}^{n}\int_{(j-1)h}^{jh}\bar{\rho}\phi(x)W(x)dx = 0 \tag{6}$$

where j represents the number of element. Each integral equation can be used for each element to determine the stiffness matrices and mass matrices. The transverse deflection, $W(x)$ is assumed as

$$W(x) = \mathbf{L}^T\bar{\mathbf{w}}_j,\quad \frac{d}{dx}W(x) = \left(\frac{1}{l_{elm}}\right)\mathbf{L}'^T\bar{\mathbf{w}}_j,\quad \frac{d^2}{dx^2}W(x) = \left(\frac{1}{l_{elm}}\right)^2\mathbf{L}''^T\bar{\mathbf{w}}_j \tag{7}$$

where $\mathbf{L}$ and $\mathbf{w}$ are the shape function and nodal vector, respectively, and can be expressed as

$$\mathbf{L} = \left[3\xi^2-2\xi^3 \quad l_{elm}\left(\xi^2-\xi^3\right) \quad 1-3\xi^2+2\xi^3 \quad l_{elm}\left(-\xi+2\xi^2-\xi^3\right)\right]^T \tag{8}$$

$$\bar{\mathbf{w}}_j = \left[W_{j-1} \quad \theta_{j-1} \quad W_j \quad \theta_j\right]^T \tag{9}$$

$$(j-1)l_{elm} \le x \le jl_{elm},\quad \xi = j - \frac{x}{l_{elm}},\quad 0 \le \xi \le 1 \tag{10}$$

By setting the test function equal to $W(x)$ for each element, and using the assumption (10), equation (6) can be written as

$$\sum_{j=1}^{n}\bar{\mathbf{w}}^T k_j\bar{\mathbf{w}} - \omega^2\sum_{j=1}^{n}\bar{\mathbf{w}}^T m_j\bar{\mathbf{w}} = 0,\quad k_j = \frac{\bar{I}}{h^3}\int_0^1\mathbf{L}''\mathbf{L}''^T d\xi,\quad m_j = h\bar{\rho}\int_0^1\mathbf{L}\mathbf{L}^T d\xi \tag{11}$$

The k_j represents the stiffness matrix and m_j describes the mass matrix for one beam element. By determining the integrals of (11), these matrices can be obtained as

$$k_j = \frac{\bar{I}}{l_{elm}^3}\begin{bmatrix}12 & 6l_{elm} & -12 & 6l_{elm}\\ 6l_{elm} & 4l_{elm}^2 & -6l_{elm} & 2l_{elm}^2\\ -12 & -6l_{elm} & 12 & -6l_{elm}\\ 6l_{elm} & 2l_{elm}^2 & -6l_{elm} & 4l_{elm}^2\end{bmatrix} \tag{12}$$

$$m_j = \frac{l_{elm}\bar{\rho}}{420}\begin{bmatrix}156 & 22l_{elm} & 54 & -13l_{elm}\\ 22l_{elm} & 4l_{elm}^2 & 13l_{elm} & -3l_{elm}^2\\ 54 & 13l_{elm} & 156 & -22l_{elm}\\ -13l_{elm} & -3l_{elm}^2 & -22l_{elm} & 4l_{elm}^2\end{bmatrix} \tag{13}$$

By considering the number of nodes, the stiffness and the mass matrices for each element can be assembled together and synthesized into the global stiffness and mass matrices. Thus, the eigen-value problem of (11) can be represented as (14) [19].

$$K\bar{\mathbf{w}} = \omega^2 M\bar{\mathbf{w}} \tag{14}$$

where K and M are global stiffness and mass matrices for an arbitrary beam. These matrices are valid for the part of the beam with symmetric piezoelectric patch. However, these matrices are applicable for the beam without piezoelectric patch by considering $\bar{I}$ and $\bar{\rho}$ equal to original beam E × I and ρ × A. Finite element formulation is utilized to write in-house MATLAB program. The actuator distributed moment on the beam element can be obtain through the virtual work. The virtual work done by moment is expressed as follows

$$\delta W = \int_{(j-1)l_{elm}}^{jl_{elm}} M_{ac}(x,t)\bullet\delta\left(\frac{\partial^2 w}{\partial x^2}\right)dx \tag{15}$$

By substituting (7) for actuator moment into (15), equation (16) is obtained

$$\delta W = -bE_{ac}\left(\frac{h_{bm}+h_{ac}}{2}\right)d_{31}V(t)\bullet\delta\left(\frac{\partial w}{\partial x}\right)\Bigg|_{(j-1)l_{elm}}^{jl_{elm}} \tag{16}$$

Substituting (7) into (6) and changing the integral band in order to local coordinates, (10), equation (16) can be rewritten as

$$\delta W = -bE_{ac}\left(\frac{h_{bm}+h_{ac}}{2}\right)d_{31}V(t)\bullet\delta\left(\bar{\mathbf{w}}^T\right)\mathbf{L}'\Big|_0^1,\quad \delta W = -\delta\left(\bar{\mathbf{w}}^T\right)bE_{ac}\left(\frac{h_{bm}+h_{ac}}{2}\right)d_{31}V(t)\left[0 \quad 1 \quad 0 \quad -1\right]^T \tag{17}$$

where first term after equality shows the transverse variation and, thus, the actuation force expresses as

$$\left\{P_{ac}\right\} = bE_{ac}\left(\frac{h_{bm}+h_{ac}}{2}\right)d_{31}V_{ac}(t)\left[0 \quad 1 \quad 0 \quad -1\right]^T,\quad \left\{P_{ac}\right\} = \left\{f_{ac}\right\}V_{ac}(t) \tag{18}$$

f_{ac} is the force vector of piezo-actuator, which maps the control voltage to the structure.

IV. Response of the System

For a Multi-degree of freedom (MDOF) system, the time response of the system can be expressed as matrix form[11, 20].

$$[m]\{\ddot{\eta}(t)\} + [c]\{\dot{\eta}(t)\} + [k]\{\eta(t)\} = \{P_{ac}\} + \{P_{ex}\} \tag{19}$$

Where m is the mass matrix, k is the stiffness matrix, $P_{ac}$is the actuation force and P_{ex} is the external force. c is damping matrix, which can be expressed as proportional damping, which is typically mentioned as Rayleigh damping. For damped structure, the closed forms of solution are not generally feasible. However, the idealized solution of damped can be assumed by utilizing classical damping. Classical damping is usually divided into category, Rayleigh damping and Caughey damping. Rayleigh damping method is widely used in beamlike structures, where shows acceptable model of structure damping. In this study, Rayleigh damping method is utilized which represents a linear combination of mass and stiffness matrices[21]

$$[c] = \alpha[m] + \beta[k] \tag{20}$$

whereα and β are known proportional Rayleigh coefficients respect to mass and stiffness matrices, respectively. α and β are defined as

$$\alpha = 2\xi_i\left(\frac{\omega_i \times \omega_j}{\omega_i + \omega_j}\right),\ \beta = 2\xi_i\left(\frac{1}{\omega_i + \omega_j}\right) \tag{21}$$

ξ is the modal viscous damping coefficient, which correspond to un-damped natural frequency, ω_i. The damping coefficient is the dynamic property of material, which cannot be determined theoretically. In this study, a uniform damping coefficient of 0.5% is assumed taking into account the results obtained from to previous experimental investigations and dynamic properties of metals [4, 17, 21, 22].

A. Modal Order Reduction

To facilitate the solution of the dynamical system (19), which involves very large matrices, resort is made to order reduction method. The concept is to estimate the high dimensional state space by using an appropriate low dimensional subspace to obtain a smaller system with approximately similar properties. To design the linear controller, first and second natural modes of the beam vibration are considered, since the other natural modes exhibit insignificant effect in comparison to the first two modes. In this regard, modal order reduction technique is utilized to reduce the large number of order of the system, which is obtained by finite element solution. Thus, first the coordinate of the system is reduced by considering the first two modes.

$$\begin{aligned} &\{\eta(t)\} = [T]_{n\times 2}\{g(t)\}_{2\times 1} \\ &\{g(t)\} = \begin{Bmatrix} g_1 \\ g_2 \end{Bmatrix} \end{aligned} \tag{22}$$

where$T$ is the reduction matrix of eigen-vectors based on first two modes, and g is the reduced coordinates. By substituting (22) into (19) and multiplying by $[T]^T$, the reduced order of the transfer function of the system can be obtained.

$$\begin{aligned} &[T]^T[m][T]\{\ddot{g}(t)\} + [T]^T[c][T]\{\dot{g}(t)\} \\ &+ [T]^T[k][T]\{g(t)\} = [T]^T\{f_{ac}\} + [T]^T\{f_{ex}\} \end{aligned} \tag{23}$$

where mass damping and stiffnessmatrices and force vectors can be defined as follows

$$\begin{aligned} &[\hat{m}]_{2\times 2} = [T]^T_{2\times n}[m]_{n\times n}[T]_{n\times 2}, \\ &[\hat{c}]_{2\times 2} = [T]^T_{2\times n}[c]_{n\times n}[T]_{n\times 2}, \\ &[\hat{k}]_{2\times 2} = [T]^T_{2\times n}[k]_{n\times n}[T]_{n\times 2}, \\ &\{\hat{f}_{ac}\}_{2\times 1} = [T]^T_{2\times n}\{f_{ac}\}_{n\times 1}, \\ &\{\hat{f}_{ex}\}_{2\times 1} = [T]^T_{2\times n}\{f_{ex}\}_{n\times 1}, \end{aligned} \tag{24}$$

Thus, equation (23) can be rewritten as

$$[\hat{m}]\{\ddot{g}(t)\} + [\hat{c}]\{\dot{g}(t)\} + [\hat{k}]\{g(t)\} = \{\hat{f}_{ac}\} + \{\hat{f}_{ex}\} \tag{25}$$

B. State-Space Representation

Equation (25) is transformed to a state space vector dynamic equation for designing the state feedback control system. To express in state space, it is solved for $\ddot{g}$.

$$\{\dot{g}(t)\} = \{\dot{g}(t)\} \tag{26}$$

$$\begin{aligned} &\{\ddot{g}(t)\} = -[\hat{m}]^{-1}[\hat{c}]\{\dot{g}(t)\} - [\hat{m}]^{-1}[\hat{k}]\{g(t)\} \\ &+ [\hat{m}]^{-1}\{\hat{f}_{ac}\} + [\hat{m}]^{-1}\{\hat{f}_{ex}\} \end{aligned} \tag{27}$$

Then, X vector is introduced in order to reduce the order of (27) as

$$\begin{aligned} &\{g(t)\} = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \\ &\{\dot{g}(t)\} = \begin{bmatrix} \dot{g}_1 \\ \dot{g}_2 \end{bmatrix} = \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix}, \\ &\{\ddot{g}(t)\} = \begin{bmatrix} \ddot{g}_1 \\ \ddot{g}_2 \end{bmatrix} = \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \end{bmatrix} = \begin{bmatrix} \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} \end{aligned} \tag{28}$$

Thus, equations(26) and (27) can be represented, respectively, as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} X_3 \\ X_4 \end{bmatrix} \tag{29}$$

$$\begin{aligned} &\begin{bmatrix} \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = -[\hat{m}]^{-1}[\hat{c}]\begin{bmatrix} X_3 \\ X_4 \end{bmatrix} - [\hat{m}]^{-1}[\hat{k}]\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \\ &+ [\hat{m}]^{-1}\{\hat{f}_{ac}\} + [\hat{m}]^{-1}\{\hat{f}_{ex}\} \end{aligned} \tag{30}$$

Equations (29) and (30) can be demonstrated in state space form as

$$\begin{aligned} &\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} [0]_{2\times 2} & I_{2\times 2} \\ -[\hat{m}]^{-1}[\hat{k}] & -[\hat{m}]^{-1}[\hat{c}] \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} \\ &+ \begin{bmatrix} [0]_{2\times 1} \\ [\hat{m}]^{-1}\{\hat{f}_{ac}\} \end{bmatrix} + \begin{bmatrix} [0]_{2\times 1} \\ [\hat{m}]^{-1}\{f_{ex}\} \end{bmatrix} \end{aligned} \tag{31}$$

$$\{\dot{X}\} = [A]\{X\} + [B] + [B^*] \tag{32}$$

where$A$, B and B^* are termed as state matrix, input matrix correspond to actuator and input matrix correspond to external force, respectively. The output of the system is expressed as Y. In this study, the output of the system is sensor voltage, where it was obtained in previous part. Thus, the output can represent as

$$Y = [C]\{X\} \tag{33}$$

where$C$ is output matrix, and shown as

$$[C] = C_{sn}\left[\theta_1(x_{s2}) - \theta_1(x_{s1}) \quad \theta_2(x_{s2}) - \theta_2(x_{s1}) \quad 0 \quad 0\right] \tag{34}$$

whereθ is derivative of displacement and x_s is the location of piezoelectric sensor on the beam. The benefits of state space approach are in the formulation of the appropriate control to obtain the desired output.

V. Control Strategies

In this investigation, two linear control methods are applied to suppress the vibration of the beam. The state space model consisting of the first two natural modes of the system is utilized to design the controller. First, PID control method is considered, which is a well-known classical control method. Then, LQR modern control is utilized to design an optimal controller in order to compare with PID control.

A. PID Control

In this study, Proportional-Integral-Derivative (PID) control is considered for controlling the flexible beam structure. PID is a well-known control tool, due to its robustness andsimplicity. The proportional feedback constant, P, controls the natural frequency of the system, and therefore control the amplitude of vibration. The integral constant, I, set the necessary adjustment for the damping, or energy dissipation of the system. The combination of proportional and integral control action gives the controller a way to minimize steady state error, while having the ability to minimize the effects of disturbances to the system. Proportional and integral constants manipulate past control error and cannot prognosticate the future control error. Thus the derivative constant, D, is proportional to the change in the error. In other words, it manipulates the speed or response of the controller. The convenient selections of the PID constants are a key aspect in the success of executing the PID controller. The transfer function of the PID controller is given by[11]:

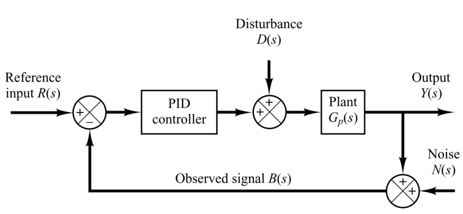


Fig. 2. Closed-loop system with PID control block diagram

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \tag{35}$$

whereK_p is the proportional gain, K_i is the integral gain and K_d is the derivative gain. Fig. 2shows the block diagram of PID control system.

B. LQR Control with Observer

Linear Quadratic Regulator (LQR) is an optimal control method, which provides a symmetric way to determine the state feedback control matrix11. For controlling the beam vibration, all variables for LQR control are not available. An observer is necessary to design an estimator for the unavailable feedback values, since it can estimate the unavailable variables. In order to control the system with observer, it should be controllable and observable. The definition of system controllability, system observability, observer and LQR are elaborated subsequently in the following, respectively.

A system is called controllable, if a system, with regard to unconstrained input control vector, can be transferred from any initial value $\mathbf{X}(t_0)$ at time t_0 to the any specified value in a specific time $t_0 \le t \le t_1$. The controllability matrix is expressed as[11]

$$\left[\mathbf{B} \mid \mathbf{AB} \mid \ldots \mid \mathbf{A}^{n-2}\mathbf{B} \mid \mathbf{A}^{n-1}\mathbf{B}\right]_{n\times n} \tag{36}$$

The system is controllable, if and only if the controllability matrix (36) is a full rank matrix (rank of n); in other words, each vectors of the matrix (36) should be linearly independent. In this regard, MATLAB program can be utilized to determine the controllability matrix.

One system is called observable, if every state vector $\mathbf{X}(t_0)$ can be obtained from the observation output in a specific time, $t_0 \le t \le t_1$. In the control theory[11], the observability matrix of a system is shown as

$$\left[\mathbf{C} \mid \mathbf{CA} \mid \ldots \mid \mathbf{CA}^{n-2} \mid \mathbf{CA}^{n-1}\right]^T \tag{37}$$

One system is observable, if and only if the observability matrix has n linearly independent vectors, or it is full rank matrix. Same as controllability, MATLAB program is applicable to determine observability matrix of a system and check its rank.

1) Observer

Following the requirement of observer, the observer is designed based on pole placement method. By measuring the output of the system and control variables, it determines the estimated variables. The notation of $\hat{\mathbf{X}}$and $\hat{Y}$ is used to assign the state vector of observer and the estimated output of observer, respectively. The observer gain, K_{ob}, can be obtained by pole placement method[11]. Thus, the equation of full state observer is expressed as

$$\dot{\hat{\mathbf{X}}} = [A]\hat{\mathbf{X}} + [B]f + K_{ob}(Y - \hat{Y}) \tag{38}$$

where$A$ and B are state matrix and actuation matrix, which are same as the state space (32). f and Y are input and output of the system, respectively, where expressed as

$$Y = \mathbf{CX}$$
$$\hat{Y} = \mathbf{C}\hat{\mathbf{X}} \quad (39)$$
$$f = V_{sn}$$

By substituting (39) into (38), it can be rewritten as

$$\dot{\hat{\mathbf{X}}} = [A - K_{ob}C]\hat{\mathbf{X}} + [B]f + K_{ob}Y$$
$$\dot{\hat{\mathbf{X}}} = [A_{ob}]\hat{\mathbf{X}} + [B \quad K_{ob}]\begin{Bmatrix} f \\ Y \end{Bmatrix} \quad (40)$$
$$\dot{\hat{\mathbf{X}}} = [A_{ob}]\hat{\mathbf{X}} + [B_{ob}]\begin{Bmatrix} f \\ Y \end{Bmatrix}$$

Equation (40) is illustrated in the block diagram form in Fig. 3.

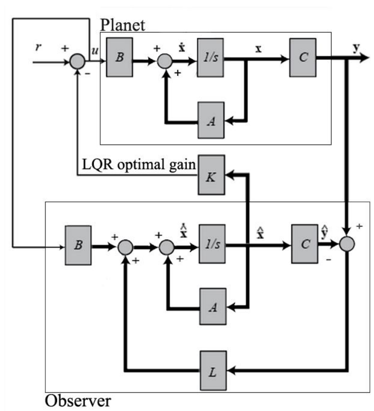


Fig. 3. Block diagram of closed-loop system with observer

2) LQR Optimal Gain

LQR control provides an approach to calculate the state feedback gain of the control system[11]. The system equation is given as

$$\dot{\mathbf{X}} = [A]\mathbf{X} + [B]\mathbf{f} \quad (41)$$

The optimal actuator input (control vector) can be determined as

$$\mathbf{f}(t) = -K_{lqr}\mathbf{X}(t) \quad (42)$$

The state feedback gain K_{lqr} is optimized to minimize the following objective function.

$$J = \int_0^\infty \left(\mathbf{X}^T\mathbf{QX} + \mathbf{f}^T\mathbf{Rf}\right)dt \quad (43)$$

where **Q** and **R** are the constant weighting matrices, which are real symmetric and positive-definite matrices. **Q** and **R** can be estimated by experiments; however, assigning **Q** large with regard to **R** represents that the response attenuation has more weight than the control effort and conversely. In LQR control, the minimized control gain is expressed as

$$K_{lqr} = \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} \quad (44)$$

where **P** is the unique and positive solution of the well-known *Riccati* equation [11].

$$\mathbf{A}^T\mathbf{P} + \mathbf{PA} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \quad (45)$$

For multi degree of freedom system, the solution of (45) for P is difficult to achieve; however, it can be solved numerically. MATLAB software has a built-in command to solve *Riccati* (45) and therefore obtain K_{lqr} and ***P***. LQR controller with observer is performed in MATLAB/SIMULINK.

VI. Results and Discussion

An Aluminum beam with three different configurations is considered here.The properties of Aluminum beam can be obtained from established references and experimental work[17]; these are tabulated in Table I.The natural frequencies and natural modes for each case study are obtained through the procedure discussed in previous section. The PID controller and LQR controller are utilized to manipulate the vibration of the beam, where controllers are designed using MATLAB/SIMULINK. The results are obtained by assuming 1 *N* impulse for the duration of 0.001 second at the tip of the beam. If the actuator voltage exceeds the maximum operating voltage of the piezoelectric material, the latter may lose its polarization and piezoelectricity property. In this regard, the input control voltage is limited to ± 90 volts, which is much less than maximum operating voltage of PZT and PVDF. Since the classical method for determining PID coefficients is not applicable here, these coefficients are obtained from available experimental, and tabulated in Table II for each case study. In the present work, the LQR controller with observer is designed and simulated to control the vibration of the system. The controllability and observability of systems were checked in first step, where state space forms of the assumed beams were all controllable and observable. The observer control method is based on pole-placement, thus, new poles should be chosen with respect to the poles of the system[11]. The most conventional approach is to obtain new poles based on experience.

TABLE I. PROPERTIES OF ALUMINIUM AND PIEZOELECTRIC MATERIALS[17, 22]

	Aluminum	***PZT5-H***	***PVDF***
Modulus of Elasticity (GPa)	71	61	2
Density (Kg/m^3)	2710	7500	1780
Width (m)	0.014	0.014	0.014
Thickness (mm)	0.66	0.75	0.11
Length(m)	0.319	-	-
Strain Constant d_{31}	-	-171×10^{-12}	23×10^{-12}
Voltage Constant g_{31}	-	0.0114	0.216

TABLE II. PID COEFFICIENTS THAT OBTAINED BY EXPERIMENT FOR EACH CASE STUDY

PID Coefficients	***Case Study I***	***Case Study II***	***Case Study III***
Proportional K_p	700	25	20
Integral K_I	675	22.5	18
Derivative K_D	900	30	24

To determine the LQR optimal gain, first, *Q* and *R* should be specified. *Q* and *R* are obtained to find the best settling time by experiment. In this study, *R* is only one number and *Q* is a square diagonal matrix in which the entries of the main diagonal are all one and multiplied by a coefficient, *q*. *R* and *q* for each case study are presented in Table III.The results of each case study are given and discussed in the following paragraphs.

TABLE III. LQR PARAMETERS THAT OBTAINED BY EXPERIMENT FOR EACH CASE STUDY

LQR Parameters	***Case Study I***	***Case Study II***	***Case Study III***
q	50×10^{7}	10^{6}	9×10^{5}
R	10^{-4}	10^{-3}	10^{-3}

The first configuration is considered as a beam completely bonded with PVDF on the top and the bottom. The sensor is located at the tip of the beam, which is 15 *mm* long. Other PDVF patches are utilized as actuator. The length of PVDF on the upper surface is same as the length of the beam and the lower one is 304 *mm*. Schematic of case study one is illustrated in Fig. 4. The free vibration analysis of this case study is done by both analytical and numerical approaches. Properties of PVDF material is listed in Table I.

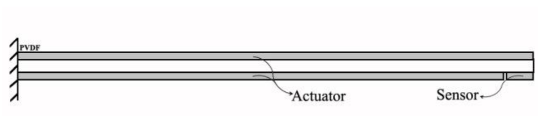
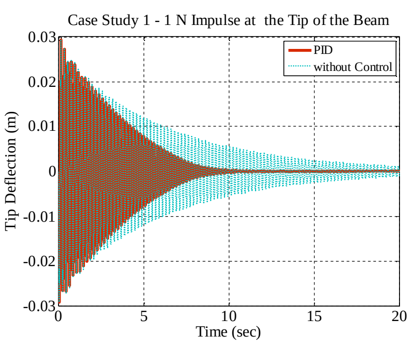


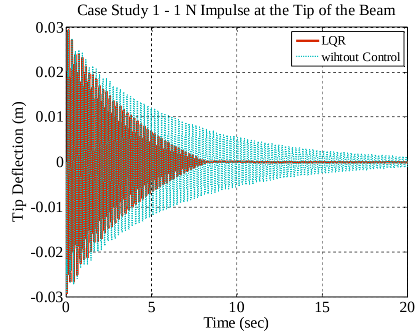
Fig. 4. Case Study one: beam with complete boned piezoelectric patch

TABLE IV. NATURAL FREQUENCIES OF CASE STUDY ONE

Natural Frequencies (Hz)	***Analytical[18]***	***FEM***	***Error%***
1st Mode	4.9501	4.9502	2.02×10^{-3}
2nd Mode	31.0197	31.0222	8.05×10^{-3}
3rd Mode	86.8618	86.8636	2.07×10^{-3}



(a)



(b)

Fig. 5. Time response of the beam, case study one (a) PID control (b) LQR control

Equation (14) is utilized to obtain the eigen-value and, consequently, the natural frequencies, via FEM and compared with analytical result [18]as shown in Table IV. The results of the FEM solution show very small error in comparison to the analytical solution.

Controlled and uncontrolled responses of the case study one is shown in Fig. 5. Time response of PID and LQR controller are independently compared with uncontrolled system. In this case, LQR shows a little better settling time in comparison to PID. Table VII expresses settling time of these controllers.

In case study two, two PZT are bonded at the base of the beam, which are 38 *mm* long. PZT sensor is located right after the piezo-actuator on the top of the beam. The length of the sensor is 15 *mm*, as shown in Fig. 6. The natural frequencies of this case study are obtained by finite element methods and validated by experimental result in a previous study[17], and expressed in Table V.

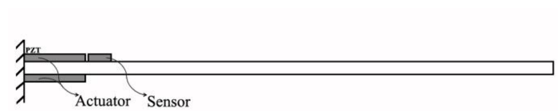


Fig. 6. Schematic of case study two

TABLE V. NATURAL FREQUENCIES OF CASE STUDY TWO COMPARED WITH AVAILABLE EXPERIMENTAL STUDY [17]

Natural Frequencies(Hz)	***Experimental Study[17]***	***FEM***	***Error%***
1st Mode	6.3	6.44	2.22
2nd Mode	38	39.45	3.81
3rd Mode	99	107.65	8.73

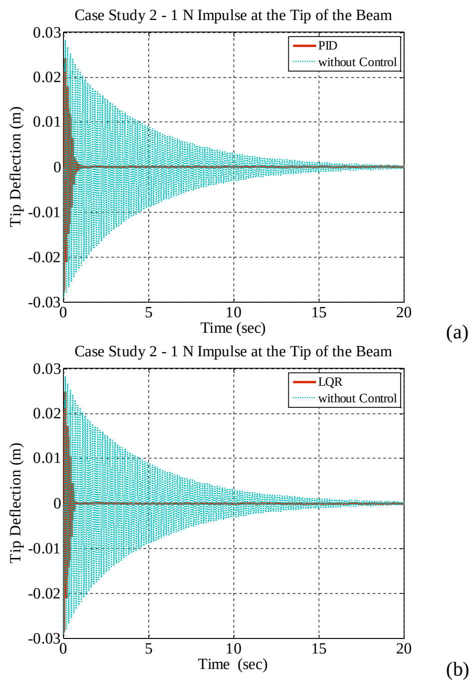


Fig. 7. Time response of the beam, case study two (a) PID control (b) LQR control

The result of PID and LQR controllers are given in Fig. 7, which exhibits significant performance. Both of them can manipulate the vibration of the system immediately; however, the results of LQR control are a little bit better than PID.

Two PZT actuators and one PVDF sensor are considered in case study three. The length of each actuator is 38 *cm* and both of them are laid on the upper surface of the beam. The sensor is located between two piezo-actuators with 2 *mm* distance from each one. The length of piezo-sensor is 15 *mm*. Fig. 8 shows the location of actuators and sensor on the beam. In this case, natural frequencies and natural modes are determined by finite element method, which are shown in Table VI.

TABLE VI. NATURAL FREQUENCIES OF CASE STUDY THREE DETERMINED BY FINITE ELEMENT METHOD

Natural Frequencies	***FEM***
1st Mode	6.87
2nd Mode	37.17
3rd Mode	98.60

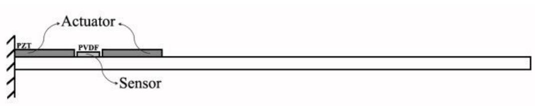


Fig. 8. Schematic of case study three

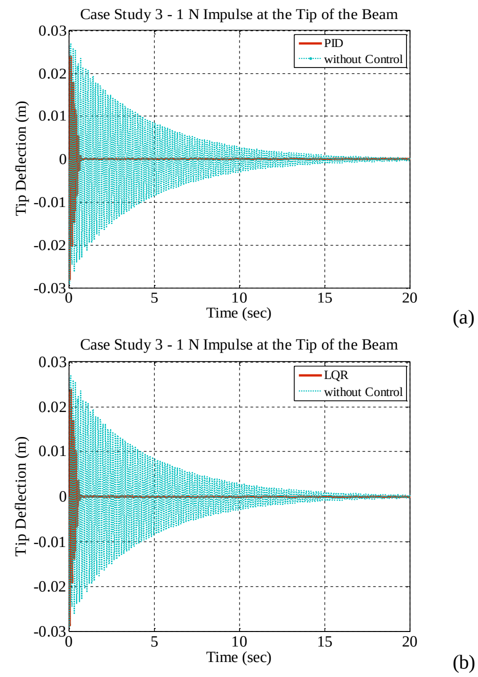


Fig. 9. Time response of the beam, case study three (a) PID control (b) LQR control

The controlled and uncontrolled systems of case study 3 are illustrated in Fig. 9. The performance of LQR is also shown to be better than that of PID. Three case studies are considered to evaluate which one has better performance in controlling the system. Settling time and root-mean-square (RMS) value of all of these case studies are exhibited in Table VII. The settling time is defined as the time taken for the response of the system to reach and stay in a range around the desired value, which is usually considered to be 2 to 5 percent of the final value. The RMS value is the square root of the arithmetic mean of the squared magnitudes of the waveform, which is a measure of the wave amplitudes.

As shown in the figures, it is obvious that the beam with PZT actuator has much better response in comparison to the beam bonded with PVDF. PZT can surpass the vibration better than PVDF, since it has higher stiffness and strain constant than PVDF. Stiffness has an effect on passive vibration of the beam, where in uncontrolled beam, the beam with PZT configuration has less settling time than beam bonded with PVDF, as exhibited in Table VII. Higher strain constant of actuator results in higher moment on the beam. Thus, the system can be controlled quickly with PZT actuator. The settling time and RMS of case study two and three are very close; however, case study three can surpass the vibration slightly better than case study two. By utilizing the PVDF sensor in the case study three, the weight of the system reduces in comparison to case study 2. Both controllers provide significant vibration suppression. For all of these case studies, LQR shows a slightly better settling time performance than PID. Case study three with LQR controller shows the best result in this study. Settling time and RMS value of it is less than the others and the weight of the beam is acceptable.

TABLE VII. Comparison of case studies and controllers

Response of the system		*Case study 1*	*Case study 2*	*Case Study 3*
Without Control	Settling Time (s)	24.29	17.70	17.43
	RMS × 10^{-5}	483	400	390
PID Control	Settling Time (s)	9.34	0.88	0.68
	RMS × 10^{-5}	414	138	135
LQR Control	Settling Time (s)	7.87	0.66	0.64
	RMS × 10^{-5}	385	134	129
Weight (g) of the Beam		9.7	10.2	9.8

VII. CONCLUSION

In this investigation, the Vibration Analysis of a Cantilevered Beam with Piezoelectric Actuator as a Controllable Elastic Structure has been elaborated by solving generic beam problem and three configurations of beam-piezoelectric elements composite beams. The active control of the vibration of an Aluminum flexible beam is studied through the bonding of the piezo-electric elements onto a beam as a smart material. Two widely utilized piezoelectric materials, PZT and PVDF, are considered for controlling the system. The dynamic of the beam bonded with piezoelectric material patches is investigated employing Euler-Bernoulli beam theory. The equation of motion of the beam is obtained through the use of Hamilton's principle. Three different configurations for the beam are selected for comparison and to gain insight into the issue. Free vibration of the first case study is determined by both analytical and finite element method. The second case study is solved by finite element and compared to an experimental work. The third one is determined by finite element only. To design the controller, state space form of each beam is formed. Two straightforward and convenient control method are investigated, PID and LQR with observer. The results thus obtained exhibit the effectiveness of the various controllers chosen and give a beneficial insight on the utilization of the stability and control of flexible beam structure.

Acknowledgments

The authors would like to thank Universiti Putra Malaysia (UPM) for granting Research University Grant Scheme (RUGS) No.9378200, and the ministry of higher education ERGS: 5527088 ; FRGS:5524250 under which the present research is carried out.

References

[1] M. Azadi, S. Fazelzadeh, M. Eghtesad, and E. Azadi, "Vibration suppression and adaptive-robust control of a smart flexible satellite with three axes maneuvering," Acta Astronautica, vol. 69, pp. 307-322, 2011.

[2] P. Gasbarri, M. Sabatini, N. Leonangeli, and G. B. Palmerini, "Flexibility issues in discrete on–off actuated spacecraft: Numerical and experimental tests," Acta Astronautica, vol. 101, pp. 81-97, 2014.

[3] M. Sabatini, P. Gasbarri, R. Monti, and G. B. Palmerini, "Vibration control of a flexible space manipulator during on orbit operations," Acta astronautica, vol. 73, pp. 109-121, 2012.

[4] S. Narayanan and V. Balamurugan, "Finite element modelling of piezolaminated smart structures for active vibration control with distributed sensors and actuators," Journal of sound and vibration, vol. 262, pp. 529-562, 2003.

[5] S. Xu and T. Koko, "Finite element analysis and design of actively controlled piezoelectric smart structures," Finite elements in analysis and design, vol. 40, pp. 241-262, 2004.

[6] C. C. Fuller, S. Elliott, and P. A. Nelson, Active control of vibration: Academic Press, 1996.

[7] N. Jalili, "Piezoelectric-based vibration control," From Macro to Micro/Nano Scale Systems Springer, 2010.

[8] H. Baruh, Analytical dynamics: WCB/McGraw-Hill Boston, 1999.

[9] L. Meirovitch, Principles and techniques of vibrations vol. 1: Prentice Hall New Jersey, 1997.

[10] R. Alkhatib and M. Golnaraghi, "Active structural vibration control: a review," Shock and Vibration Digest, vol. 35, p. 367, 2003.

[11] K. Ogata and Y. Yang, "Modern control engineering," 1970.

[12] H. Djojodihardjo, "Computational simulation for analysis and synthesis of impact resilient structure," Acta Astronautica, vol. 91, pp. 283-301, 2013.

[13] H. Djojodihardjo and A.Shokrani, "Generic Study And Finite Element Analysis Of Impact Loading On Elastic Panel Structure, Paper IAC-10.C2.6.2," presented at the 61th International Astronautical Congress, Prague, The Czech Republic, 2010.

[14] H. Djojodihardjo, P.M.Ng, and L.K.Soo, "Analysis And Simulation Of Impact Loading On Elastic Beam Structure with Case Studies, Paper IAC-09-C2.6.01," presented at the 60th International Astronautical Congress, Daejeon, Republic Of Korea, 2009.

[15] H. Djojodihardjo and I. Safari, "BEM-FEM Coupling For Acoustic Effects On Aeroelastic Stability Of Structures," CMES: Computer Modeling in Engineering & Sciences, vol. 91, pp. 205-234, 2013.

[16] M. Jafari, H. Djojodihardjo, and K. Arifin Ahmad, "Vibration Analysis of a Cantilevered Beam with Spring Loading at the Tip as a Generic Elastic Structure," presented at the Aerotech, Kuala Lumpur, Malaysia, 2014.

[17] S.-Y. Hong, "Active vibration control of adaptive flexible structures using piezoelectric smart sensors and actuators," 1992.

[18] H. Djojodihardjo and M. Jafari, "Vibration analysis of a cantilevered beam with piezoelectric actuator as a controllable elastic structure," presented at the accepted in 65 th International Astronautical Congress, Toronto, Canada., 2014.

[19] L. Meirovitch, Computational methods in structural dynamics: Springer, 1980.

[20] S. S. Rao, Vibration of continuous systems: John Wiley & Sons, 2007.

[21] A. K. Chopra, Dynamics of structures vol. 3: Prentice Hall New Jersey, 1995.

[22] A. Yousefi-Koma, "Active vibration control of smart structures using piezoelements," Carleton University, 1997.