A Parallel Robotic Mechanism Replacing a Machine Bed for Micro-Machining

Zareena Kausar  Muhammad Asad Irshad  Shaheriyar Shahid
Department of Mechatronics Engineering, Air University, Islamabad, Pakistan
corresponding author: zareena.kausar@mail.au.edu.pk

Abstract— Micromachining is meant for small accurate and precise parts production for many industrial applications. These parts vary from very simple to complicated shapes. As applications grow in complexity and shrink in size, machines need to be designed to meet the desired precision and accuracy. In this paper it is proposed to give a six degree of freedom to the bed of a machine instead to tool box which carry a heavy load. A parallel mechanism consists of six legs of variable lengths is proposed for the machine bed. The objective is to enhance accuracy and precision in micromachining in comparison to conventional machines. The paper presents a design of the machine bed, the kinematics, dynamics and a sim-mechanics model of the bed. Simulation results verify the working of a 6-DOF machine bed for micromachining.

Keywords— Parallel Mechanism; Robotics; Micromachining; Machine bed; Manufacturing.

I. INTRODUCTION

Micromachining is a process of fabrication of microstructures using geometrically determined cutting edges techniques like, drilling, milling, de-burring and slotting. There are few limitations about geometry of work piece and also 3-D structures that are manufactured. The components produced on micro machine tools are of a size in a range of centimeter or millimeter [1]. Different machine tools [2, 3] are developed for micromachining. One of such a machine tool is developed at the Fraunhofer institute, named as ‘MiniMill’, for production sciences in Aachen [4]. This is a high precision machine tool featuring one square meter of floor space. This machine tool focuses on high precision and thermal stability for achieving high accuracy [5]. Another device for micro production is developed at the National Autonomous University of Mexico [6] for the machining of micro structures of different work pieces with an accuracy of 50 micrometer. The fact that the micro-machines operate on very small parts using very small tools means that they are vulnerable to external and internal vibrations. In the presence of vibrations the cuts made and the parts created will be dimensionally inaccurate and relatively imprecise. To solve the problem, this research proposes to restrict the tool motion in a single dimension and instead replace the existing beds with a 6-DOF manipulator.

There are several types of manipulators serial and parallel. We propose a parallel manipulator, since it offers a high stiffness, high dynamics and prevents the accumulation of position errors. A parallel manipulator has received a lot of attention recently in the industry and the robotic community due to its high accuracy, high speed and high load capacity in comparison to conventional serial manipulators. Several serial links control a platform simultaneously to make a parallel manipulator. The most known configuration of parallel manipulator is hexapod used for multiple applications.

The proposed parallel mechanism for mechanical micromachining has same concept as a Stewart platform does [7]. The tool is fixed and all type of motions is being created by bed. With other conventional robots, adjustments to these stations often require shutting down the entire line to reset tool for a different part to be done. With the flexibility of a Parallel Mechanism tool might never need to be moved again. Imagine a process where, when a new part is accessible into the station, motion of platform can align each part individually.

The mechanism of the movable bed is presented in next section. Kinematics of the mechanism is described in the form of inverse kinematics along with the position and velocity analysis. A dynamics model of the mechanism is developed in section-IV and the Sim-mechanic model follows it. The model is used to simulate the results which are presented as results.

II. MECHANISM DESIGN AND ANALYSIS

Mechanical design is presented which may be used to determine the performance measures such as, positioning accuracy, repeatability and freedom from vibration. The Stewart platform mechanism is a parallel kinematic structure that can be used as a basis for controlled motion with 6 degrees of freedom, such as manufacturing process and precise manipulative tasks. Stewart platform are resourceful and effective solutions to complex motion applications that require high load capacity and accuracy in up to six independent axes [8].

The proposed Stewart platform like manipulator for this research consists of a rigid moving plate, connected to a fixed base through six independent legs [9]. These legs are identical kinematics chains, coupling the moveable upper platform and the fixed lower platform. The position and orientation of the end-effector is controlled by the lengths of six linear actuators which may be driven by servo motors that connect it to the base. At the base end, each actuator is connected by a two-degree-of-freedom universal joint. At the end-effector, each actuator is attached with a three-degree-of-freedom ball-and-socket joint. Thus, length of the legs is variable and they can be controlled separately to control the motion of the moving
platform. It exhibits characteristics of closed-loop mechanisms.

![Figure 1: Schematic model of the system](image)

### A. Workspace

The workspace of the manipulator is defined in two ways [9]: *reachable workspace* and *dexterous workspace*. The reachable workspace is the collection of all points \( \{x, y, z\}^T \) that can be reached by the manipulator in any orientation. The dexterous workspace is the collection of all points that can be reached by manipulator in all orientations consequently; the dexterous workspace is a subset of reachable workspace. For the parallel manipulator the dexterous workspace is null, since this cannot reach all orientations at any position in the reachable workspace. Here we define the dexterous workspace as collection of all points that can be reached by the manipulator in all orientations. Consequently, the dexterous workspace is a subset of reachable workspace. The workspace of the parallel manipulator is described based on assumptions that there is no actuator limitation, leg interference and singularities.

### III. KINEMATICS ANALYSIS

To analyze kinematics of the mechanism, the manipulator motion is studied without regard to the forces that cause it. Within this analysis the position and the velocity of the manipulator are studied. This study of kinematics of manipulators also refers to all the geometrical and time-based properties of motion [5]. Here in this article, we considered inverse kinematics of the parallel manipulator. The inverse kinematics analysis of parallel manipulators gives the actuator displacement from the given position and orientation of a movable platform. The solution is unique, and can be simply determined. The forward kinematics analysis determines the position and orientation of moveable platform for the given actuator displacement which is not recommended for this study. The reason was that the solution becomes complicated because the problem involves system of higher order nonlinear equations, as in [10].

### A. Inverse kinematics

In inverse kinematics analysis given the desired position and orientation, we computed the set of joint angles. Inverse kinematics derived in this article is based on [1, 11]. The coordinate frame \( A(x, y, z) \) is attached to the fixed base and the coordinate frame \( B(x', y', z') \) is attached to a moving platform. Furthermore, a local coordinate frame \( C(x_i, y_i, z_i) \) is attached to each limb such that its origin is at point \( A_i \), the \( z_i \) axis points from \( A_i \) to \( B_i \), the \( y_i \) axis is parallel to the cross product of two unit vectors defined along the \( z_i \) and \( z \) axis and the \( x_i \) axis is defined by right-hand rule. For the convenience, the origin of the frame \( B \) is located at mass center \( P \) of the moving platform. The location of moving platform is described by a position vector \( p \) and rotation matrix \( A^R_B \) as shown in Fig. 2.

![Figure 2: Schematic diagram of the parallel manipulator bed](image)

The rotation matrix is defined by roll, pitch and yaw angles. These are a rotation of \( \phi_i \) about the fixed \( x \)-axis, a rotation of \( \phi_y \) about the fixed \( y \)-axis and a rotation of \( \phi_z \) about the fixed \( z \)-axis. The rotation matrix is \( (1) \).

\[
A^R_B = \begin{pmatrix}
C_{\phi_x}C_{\phi_y} & C_{\phi_x}S_{\phi_y}S_{\phi_z} - S_{\phi_x}C_{\phi_z} & C_{\phi_x}S_{\phi_y}C_{\phi_z} + S_{\phi_x}S_{\phi_z} \\
S_{\phi_y}S_{\phi_z} + C_{\phi_y}S_{\phi_z} & S_{\phi_x}C_{\phi_y}C_{\phi_z} + S_{\phi_y}S_{\phi_z} & -S_{\phi_x}C_{\phi_y}S_{\phi_z} + C_{\phi_x}C_{\phi_z} \\
-S_{\phi_y}C_{\phi_z} & S_{\phi_x}S_{\phi_y}C_{\phi_z} - C_{\phi_y}S_{\phi_z} & S_{\phi_x}S_{\phi_y}S_{\phi_z} + C_{\phi_y}C_{\phi_z}
\end{pmatrix}
\]

Where \( S_{\phi_i} = \sin(\phi_i), C_{\phi_i} = \cos(\phi_i), \quad i = x, y, z \)

### B. Position analysis

From Fig. 2, a vector loop equation can be written for each limb as

\[
a_i + d_i s = p + b_i
\]

Where \( a_i = [a_{ix}, a_{iy}, a_{iz}] \) is a position vector of a ball joint \( A_i \) with respect to the fixed frame \( A \).
moving links from the independent Cartesian velocities of the platform $v_p$, $v_\phi$, $v_\psi$, $w_x$, $w_y$, $w_z$. The latter three scalar quantities are the components of the angular velocity of the moving platform $\omega$. Let $p = [p_x, p_y, p_z]^T$ be a position vector of moving platform $\dot{p} = [v_p, v_\phi, v_\psi]^T$ is a velocity vector of moving platform $\omega = [\omega_x, \omega_y, \omega_z]^T$ is an angular velocity vector of moving platform. Now $\mathbf{b}_i$ is written in term of angular velocity vector of the $i$th leg as

$$\mathbf{b}_i = \mathbf{d}_i + \mathbf{\omega}_i \times \mathbf{d}_i$$

Eq. (7) is expressed in matrix form as (8).

$$\mathbf{C}_{bi} \lambda_{bi} = \mathbf{b}_i$$

where $i = 1, 2, 3, 4, 5, 6$

and

$$\mathbf{d}_i = \begin{bmatrix} \mathbf{d}_{i\phi} \\ \mathbf{d}_{i\psi} \\ \mathbf{d}_{i\theta} \end{bmatrix}, \quad \mathbf{\omega} = \begin{bmatrix} \mathbf{\omega}_x \\ \mathbf{\omega}_y \\ \mathbf{\omega}_z \end{bmatrix}$$

Eq. 8 is expression of angular velocity and can be solved for $\lambda_{bi}$ which leads to the determination of $\mathbf{d}_i, \mathbf{\phi}_i$ and $\mathbf{\theta}_i$. Once these quantities are known, the computation of velocities of $i$th leg is straightforward.

$$\mathbf{\lambda}_{bi} = (\mathbf{C}_{bi})^{-1} \mathbf{b}_i$$

where $i = 1, 2, \ldots, 6$

IV. DYNAMIC ANALYSIS

In order to develop the dynamic equations of the Stewart platform manipulator, the entire system is divided in two parts: one is moving platform and the other is legs. The kinetic and potential energies for the both of these parts are figured out and the dynamic equations are derived using these energies.

A. Kinetic and potential energies of moving platform

The kinetic energy of moving platform has translation and rotational motion energies about three orthogonal axis ($X$, $Y$, $Z$). The translation energy occurring because of the translation motion of the center of mass of moving platform and is defined by (12).
\[ K_{\text{trans}} = 0.5 m [\phi_x^2 \phi_y^2 \phi_z^2] \quad (10) \]

Where \( m \) is mass of the moving platform.

For rotational motion of the moving platform around its center of mass, the rotational kinetic energy is as given in (11).

\[ K_{\text{rot}} = 0.5 [b_i^T I_{\text{rot}} b_i] \quad (11) \]

In (11) \( I_{\text{rot}} \) and \( b_i \) are the rotational inertia mass and angular velocity of the moving platform, respectively. Angular velocity \( b_i \) is defined in eq. (8) and \( I_{\text{rot}} \) is as follows:

\[
I_{\text{rot}} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}
\]

As a result, the total kinetic energy of moving platform is given by

\[ K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2} m [\phi_x^2 \phi_y^2 \phi_z^2] + \frac{1}{2} [b_i^T I_{\text{rot}} b_i] \quad (12) \]

The potential energy of moving platform is given in (13).

\[ P = \begin{bmatrix} 0 & 0 & m_{sp} g & 0 & 0 & 0 \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix} \quad (13) \]

Where \( g \) is a gravitational force.

B. Kinetic and potential energies of legs

Each leg consists of two parts: the moving part connected to moving platform through ball joint and the fixed part connected to fixed base through universal joint. As shown in Fig. 4, the center of mass is \( G_i \) for each part of leg (where \( i=1,2, \ldots, 6 \) ) and \( G \) denotes the center of mass of fixed part. \( m_1 \) and \( l_1 \) are the mass and the length of the fixed part and \( \delta \) is the distance between \( B_i \) and \( G \). For the moving part of leg, \( G_{2i} \) express its center of mass. \( M_2 \) and \( l_2 \) are the mass and length of the moving part.

The length of leg is assumed to be constant. The total kinetic energy of a leg, \( L_i \), is calculated as (14).

\[ K_{L_i} = K_{L_i,\text{trans}} + K_{L_i,\text{rot}} = \frac{1}{2} m_1 (\dot{m}_1 + m_2) \left[ \dot{L}_i \right]^2 + \frac{1}{2} [L_i^T I_{\text{rot}} L_i] \quad (14) \]

Where \( K_{L_i} \) is \( (6 \times 6) \) spatial inertia matrix of the leg \( (Li) \). By using this vector calculation of leg velocity is as \( \dot{L}_i = V_{\eta} U_i \).

Total potential energy of leg is

\[ P_{\text{leg}} = (m_1 + m_2) g \sum_{i=0}^{n} \left[ \frac{1}{L_{2i}} + \frac{1}{L_{2i-1}} \right] \frac{2m_3}{m_4 + m_5} (P_x + Z_{\eta}) \quad (15) \]

Figure 4: A leg of the parallel manipulator bed

C. Dynamic Equations of Motion of Parallel Manipulator Bed

The generalized dynamic model of the Stewart platform [13] is given as (16).

\[ T = J_p^T F_p + \sum_{i=1}^{n} \left( \frac{\partial F_i}{\partial \phi} \right) T_i (H_i) \quad (16) \]

With:

\( F_p \) is the total forces and momentum on the platform \( J_p \) is the \((6 \times n)\) kinematic Jacobian matrix of the robot

The Jacobian matrix is computed following [10], which writes the platform velocity (translational and angular) as function of active joint velocities \( V_p = J_p \phi' \).

\( H_i \) is the dynamic model of \( i \)th leg, it is a function of \( (p \:\: \: \: p' \:\: \: P' \:\: \: \phi') \) which can be obtained in terms of the platform location, velocity and acceleration, using the inverse kinematic models of the legs. Any one of methods presented in [14-16] may be used to calculate these elements.

For a general case, where the platform has 6 degrees of freedom \( F_p \) is calculated using Newton-Euler equation [14, 15].

\[ F_p = I_s \left[ \begin{bmatrix} V - g \omega^2 \\ \omega \times \omega M_s \end{bmatrix} \right] + \left[ \omega \times \left( \omega \times \left[ I_p \times \omega \right] \right) \right] \quad (17) \]

Where \( I_s \) is \((6 \times 6)\) spatial inertia matrix of the platform.

\( I_p \) \((3 \times 3)\) inertia matrix of the platform around the origin of the platform.

\( M_s \) is \((3 \times 1)\) vector of first moments of the platform around the origin of the platform.

\( V \) the velocity vector of moving platform

\( \omega \) angular velocity vector of moving platform

\( \omega' \) angular acceleration of moving platform
V. MODELING, CONTROL AND SIMULATION

In this section a model of the system described in a software, controller used for simulations and simulation results are presented.

A. The Model

The model is presented in Simmechanics, a tool of Matlab®. The model shown in Fig. 5 is divided into subsystems: Plant subsystem; Controller subsystem. The plant subsystem consists of the parallel manipulator bed along with necessary actuators and sensors while a controller subsystem controls the motion of the parallel manipulator bed through a predefined motion profile with actuation signals. The controller keeps the actual motion close to the reference motion via sensor-actuator feedback.

Figure 5: Block Diagram of Simmechanic Model of the bed

A reference trajectory is given to the controller through leg trajectory block. The reference trajectory provided for use in the simulations for this study is a sinusoidal function of time.

A sinusoidal trajectory is selected, shown in Fig. 6, to define the rotational as well as translational degrees of freedom. Any kind of trajectory can be designed and implemented for this sub-subsystem.

Figure 6: Subsystem of Desired Trajectory Block

B. The Controller

A PID controller is used to track the reference trajectory. PID stands for proportional, derivative and integral controller. The selection of PID for this research was due to its simplicity in implementation. The simplest implementation of a trajectory control is to apply forces to the plant proportional to the motion error. The error is measured through Joint Sensors for which separate joint sensor blocks are used in the model.

A PID control law is a linear combination of a variable detected by a sensor, its time integral, and its first derivative. The PID controller uses the leg position errors $E_r$ and their integrals and velocities for the proposed parallel manipulator bed. The control law for an $r^{th}$ leg has the form:

$$F_{act,r} = K_p E_r + K_i \int_0^t E_r dt + K_d \frac{dE_r}{dt}$$

The controller applies the actuating force $F_{act,r}$ along the leg:

- If $E_r$ is positive, the leg is too short, and $F_{act,r}$ is positive (expansive).
- If $E_r$ is negative, the leg is too long, and $F_{act,r}$ is negative (compressive).
- If $E_r$ is zero, the leg has exactly the desired length, and $F_{act,r}$ is zero.

The real, nonnegative $K_p$, $K_i$, and $K_d$ are, respectively, the proportional, integral, and derivative gains that modulate the feedback sensor signals in the control law:

- The first term is proportional to the instantaneous leg position error or deviation from reference.
- The second term is proportional to the integral of the leg position error.
- The third term is proportional to the derivative of the leg position error.

The result is $F_{act,r}$, the actuator force applied by the controller to the legs. The proportional, integral, and derivative terms tend to make the leg’s top attachment points $p_{t,r}$ follow the reference trajectories by suppressing the motion error.

C. Simulation Results

For simulations the leg lengths were assumed to be same initially. The reference trajectory is taken as sinusoidal wave and corresponding block is added to the model. This is the reference trajectory each point on the moving bed is desired to follow. The dynamic simulations were performed whereas a couple of clips are shown in Fig. 7a and 7b.

The difference in figures depicts the stretch or compression in leg lengths which the robotic mechanism performed to achieve the desired points on the moving bed. The blue traces on the top of the bed shows the traces of path the bed followed. These results confirms that the proposed mechanism for the bed is capable of following the complex trajectory expected as the contours on the work piece to be machined.

The error in the positions is plotted as shown in Fig. 8. These are given in mm. The maximum error reaches to 5 micrometer which verifies the suitability of the design for micro machining operations. In order to increase the accuracy
VI. CONCLUSION

In this research we presented design, the kinematics and dynamic analysis of a parallel robotic mechanism for a micro machining bed. We used MATLAB Simmechanics for the dynamic analysis of parallel system. The model is simulated in order to verify the objective of the proposed mechanism. Simulations show promising results. The computed modeling error depicts the high accuracy of the developed model. It is concluded that the verified model of the proposed mechanism may be used for bed control and design purposes for micromachining. In future the mechanism will be developed to verify the results in real time.

REFERENCES


