Compressed Sensing Based Rotative Quantization in Temporally Correlated MIMO Channels

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***Abstract*—Channel adaptive transmission requires knowledge of channel state information at the transmitter. In temporally correlated MIMO channels, the correlation can be utilized to reduce feedback overhead and improve performance. In this paper, Compressed Sensing (CS) methods and rotative quantization are used to compress and feedback channel state information for MIMO systems as an extension work of [1]. Using simulation, it is shown that the CS based method reduces feedback overhead while delivering the same performance as the direct quantization scheme.**

I. Introduction

In modern wireless communications multiple-input multiple-output (MIMO) systems are integrated due to their advantage in improving performance with respect to many performance metrics. One of the advantages is the ability to transmit multiple streams using spatial multiplexing [2]. However, one needs channel state information (CSI) at the transmitter in order to get optimal system performance [3]. In frequency division duplexing (FDD) MIMO a dedicated feedback channel of limited capacity is usually assumed. Several limited feedback strategies are proposed using codebooks which are both known to the transmitter and receiver [4] - [9].

Temporaly correlation of wireless channels can be used to reduce the feedback requirement in limited feedback systems. One technique to reduce the feedback requirement in temporally correlated channels is to quantize the rotative change of singular vectors. For instance, differential rotation feedback is proposed in [4].

In [1], scalar quantization using adaptive range is used to utilize the temporal correlation. This paper, extends the work done in [1] by introducing the concept of Compressed sensing (CS) for rotative quantization methods. The near-sparse nature of the rotation matrices is used to reduce the feedback requirement by using compressed sensing based coding and decoding.

Recently questions like, why go to so much effort to acquire all the data when most of what we get will be thrown away? Can we not just directly measure the part that will not end up being thrown away?, that was paused by Donoho [13] and others, triggered a new way of sampling or sensing called compact ("compressed") sensing (CS). In compressed sensing (CS) the task is to estimate or recover a sparse or compressible vector $\mathbf{x} \in \mathbb{R}^N$ from a measurement vector $\mathbf{y} \in \mathbb{R}^M$. These are related through the linear transform $\mathbf{y} = \mathbf{A}\mathbf{x}$. Here, $\mathbf{x}$ is a sparse vector and $M \ll N$. In the seminal papers [13] and [15], $\mathbf{x}$ is estimated from $\mathbf{y}$, by the algorithm: min $||\mathbf{x}||_0$ such that $\mathbf{y} = \mathbf{A}\mathbf{x}$, This is a non convex NP-complete. The usual wisdom is to solve it using approximation with min $||\mathbf{x}||_1$ such that $\mathbf{y} = \mathbf{A}\mathbf{x}$, which is a convex optimization problem [13] - [17]. One of the most famous approaches is the l_1-regularized least square or LASSO and we used this eastimator as a recovery algorithm in this paper.

The concept of applying compressed sensing for limited (compressed) feedback of parameters of the channel is well known [10]- [12]. In this paper though, we combine rotative quantization and CS to reduce the feedback overhead in temporally correlated MIMO channels.

This paper is organized as follows. First we give the MIMO system model that we work with in Section II. In Section III, we review the concept of rotative quantization. In Section IV, it is shown how rotative quantization can be combined with CS to reduce the feedback overhead. Then in section V, the performance of the proposed method is shown using simulations. The last section gives conclusion and future work.

II. System Model

Considering a frequency division duplex (FDD) MIMO system consisting of N_t transmit and N_r receive antennas we assume that the channel is a flat-fading, temporally correllated channel denoted by a matrix $\mathbf{H}[n] \in \mathbb{C}^{N_r \times N_t}$ where n indicates a channel feedback time index with block fading assumed during the feedback interval. Applying Singular Value Decompostion (SVD) of $\mathbf{H}[n]$ gives $\mathbf{H}[n] = \mathbf{U}[n]\mathbf{\Sigma}[n]\mathbf{V}^H[n]$, where $\mathbf{U} \in \mathbb{C}^{N_r \times r}$ and $\mathbf{V} \in \mathbb{C}^{N_t \times r}$ are unitary matrices and $\mathbf{\Sigma} \in \mathbb{C}^{r \times r}$ is a diagonal matrix consisting of $r = \min(N_t, N_r)$ singular values.

In the presense of perfect channel state information (CSI) a MIMO system model can be given by the equation

$$\tilde{\mathbf{y}} = \mathbf{U}^H[n]\mathbf{H}[n]\mathbf{V}[n]\tilde{\mathbf{x}} + \mathbf{U}^H[n]\mathbf{n} \qquad \text{(II.1)}$$

where $\tilde{\mathbf{x}} \in \mathbb{C}^{r \times 1}$ is transmitted vector, $\mathbf{V}[n]$ is used as precoder at the transmitter, $\mathbf{U}^H[n]$ is used as decoder at the reciver, $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ denotes a noise vector whose entries are i.i.d. and distributed according to $\mathcal{CN}(0,1)$ and $\tilde{\mathbf{y}} \in \mathbb{C}^{N_r \times 1}$ is the received vector.

In practice, partial channel state information is available at the transmitter, hence we assume that only a quntized version $\hat{\mathbf{V}}[n]$ is available, Further, assuming a generalized receiver $\mathbf{R}[n]$, (II.1) becomes:

$$\tilde{\mathbf{y}} = \mathbf{R}^H[n]\mathbf{H}[n]\hat{\mathbf{V}}[n]\tilde{\mathbf{x}} + \mathbf{U}^H[n]\mathbf{n}. \tag{II.2}$$

In this paper we consider two different alternatives for $\mathbf{R}[n]$. Assuming a minimum mean square error (MMSE) approach we get

$$\mathbf{R}[n] = \left[\left(\mathbf{H}[n]\hat{\mathbf{V}}[n]\right)^H \mathbf{H}[n]\hat{\mathbf{V}}[n] + r\sigma_n^2\mathbf{I}\right]^{-1}\left(\mathbf{H}[n]\hat{\mathbf{V}}[n]\right)^H.$$

Alternetively, a Mached Filter (MF) receiver gives

$$\mathbf{R}[n] = \left(\mathbf{H}[n]\hat{\mathbf{V}}[n]\right)^H.$$

The reciver estimates the channel from pilot symbols, computes SVD, quantizes and then feedbacks $\tilde{\mathbf{V}}[n]$.

Further we assume a first-order Gauss-Markov process to model the channel variation in a channel with temporal correlation as used in [1] and [4] given by the equation

$$\mathbf{H}[n] = \rho\mathbf{H}[n-1] + \sqrt{1-\rho^2}\mathbf{G}[n] \tag{II.3}$$

where ρ is the temporal correlation and $\mathbf{G}[n] \in \mathbb{C}^{N_r \times N_t}$ denotes the innovation process having i.i.d. entries distributed according to $\mathcal{CN}(0,1)$. ρ is given by $\rho = J_0(2\pi f_d \tau)$, where $J_0(\cdot)$ is the zero order Bessel function of the first kind, τ is the channel feedback interval, and f_d is the Doppler frequency.

III. Rotation Based Quantization

In the temporally correlated environment we apply a rotation based limited feedback system for a Rayleigh flat fading MIMO channels as in [4]. The differential rotation of the precoder matrix at a time index n, $\mathbf{V}[n]$, compared to the one at a previouse time instant,$\mathbf{V}[n-1]$ is quantized. This has advantage of reducing the feedback overhead.

The precoder matrix $\mathbf{V}[n]$ can be represented equivalently as

$$\mathbf{V}[n] = \mathbf{T}[n]\tilde{\mathbf{I}}_{N_t \times r} \tag{III.1}$$

where, $\mathbf{T}[n] \in \mathbb{C}^{N_t \times N_t}$ is a matrix containing all singular vectors and $\tilde{\mathbf{I}}[n]$ is a matrix composed of the first m columns of the identity matrix $\mathbf{I}_{N_t \times N_t}$. As in [1] we assume that both reciver and transmitter has a common estimate $\hat{\mathbf{T}}[n]$ of $\mathbf{T}[n]$. Then we can define a rotation matrix

$$\mathbf{\Theta}[n] = \hat{\mathbf{T}}^H[n-1]\mathbf{V}[n] \tag{III.2}$$

where, $\mathbf{\Theta}[n] \in \mathbb{C}^{N_t \times r}$. A quantized version $\hat{\mathbf{\Theta}}[n]$ is then fedback to the transmitter where an estimate

$$\hat{\mathbf{V}}[n] = \hat{\mathbf{T}}[n-1]\hat{\mathbf{\Theta}}[n] \tag{III.3}$$

can be reconstracted.

In order to quantize $\mathbf{\Theta}[n]$ various methods can be used. Vector quantization of this matrix is used in [4], parametrization using Givens rotations is also used in [1]. Before introducing our new method, we present a baseline approach for comparison. This is Algorithm 1 below, where $\mathbf{\Theta}[n]$ is vectorized and vector quantization of the resulting vector is used. The codebook used for vector quantization consists of vectors uniformly distributed in a $N_t \times r$ dimensional unit hypersphere. We have assumed that the feedback channel is error free and the only inaccuracy comes from quantization error.

Algorithm 1 Rotative Quantization feedback using CS for temporally correlated MIMO channels

1. **Initialization:** (both transmitter and receiver)
Set $\tilde{\mathbf{T}}[0] = \mathbf{I}_{N \times N}$.

2. For each time index $n \geq 1$:
Receiver:
Obtain $\mathbf{\Theta}[n] = \hat{\mathbf{T}}^H[n-1]\mathbf{V}[n]$.
Update $\hat{\mathbf{T}}[n]$.
Perform vector quantization of $\hat{\mathbf{\Theta}}[n]$.

Transmitter: Re-construct $\hat{\mathbf{\Theta}}[n]$ from received parameters.
Obtain the estimate of the current singularvector matrix using $\hat{\mathbf{V}}[n] = \hat{\mathbf{T}}[n-1]\hat{\mathbf{\Theta}}[n]$.

To simplify the analysis we assume $N_r = N_t = N$, for the rest of the paper.

IV. Quantization using Compressed Sensing

We will now modify Algorithm 1 above by using CS instead of direct quantization of $\hat{\mathbf{\Theta}}[n]$. The approach is summarized in Algorithm 2 and Fig.1 below. The first step is to arrange the entries of $\mathbf{\Theta}[n]$ in a column vector $\mathbf{x}$. We denote this operator by $\mathbf{x} = \text{vec}(\mathbf{\Theta}[n])$. Assuming strong correlation, $\hat{\mathbf{\Theta}}[n]$ will be close to diagonal and $\mathbf{x}[n]$ sparse. Next, we apply a random fat matrix $\mathbf{A}$, which is known both to the transmitter and the reciver, to get another vector $\mathbf{y} = \mathbf{A}\mathbf{x}$, which has much less dimension than the orignal vector $\mathbf{x}$. A quantized version $\hat{\mathbf{y}}$ is sent through the feedback channel. The receiver reconstructs $\hat{\mathbf{x}}$ from the recived vector $\hat{\mathbf{y}}$ and the matrix $\mathbf{A}$ using LASSO:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \;\; ||\hat{\mathbf{y}} - \mathbf{A}\mathbf{x}||_2^2 + ||\mathbf{x}||_1. \tag{IV.1}$$

Then we apply the reverse process of the vectorization, unvec($\hat{\mathbf{x}}$), and we get $\hat{\mathbf{\Theta}}[n]$, which is an estimate of $\mathbf{\Theta}[n]$. Finally, since we are interested in estimating $\mathbf{V}[n]$, we can derive it from $\hat{\mathbf{V}}[n] = \hat{\mathbf{T}}[n-1]\hat{\mathbf{\Theta}}[n]$.

V. Results

In order to verify the proposed algorithm, we consider a 2×2 MIMO channel with a temporal correlation of 0.98. Spatial streams are assumed to be transmitted with equal power allocation in the 2×2 MIMO system. Unitary precoding is applied based on the feedback, and matched filter or MMSE equalizers are applied at the receiver.

Algorithm 2 Rotative Quantization feedback using CS for temporally correlated MIMO channels

For each time index $n \geq 1$:
Receiver:
Vectorize $\boldsymbol{\Theta}[n]$ obtained from algorithm 1, $\mathbf{x} = \text{vec}(\boldsymbol{\Theta}[n])$.
CS encoding $\mathbf{y} = \mathbf{A}\mathbf{x}$.
Quantize $\mathbf{y}$ and feedback the resulting $\hat{\mathbf{y}}$.

Transmitter:
Recover using LASSO, $\hat{\mathbf{x}} = \underset{\mathbf{x}}{\text{argmin}} \ ||\hat{\mathbf{y}} - \mathbf{A}\mathbf{x}||_2^2 + ||\mathbf{x}||_1$.
Unvectorize $\hat{\mathbf{x}}$ to get $\hat{\boldsymbol{\Theta}}[n]$.
Obtain $\hat{\mathbf{V}}[n] = \mathbf{T}[n-1]\hat{\boldsymbol{\Theta}}[n]$ (as in algorithm 1).

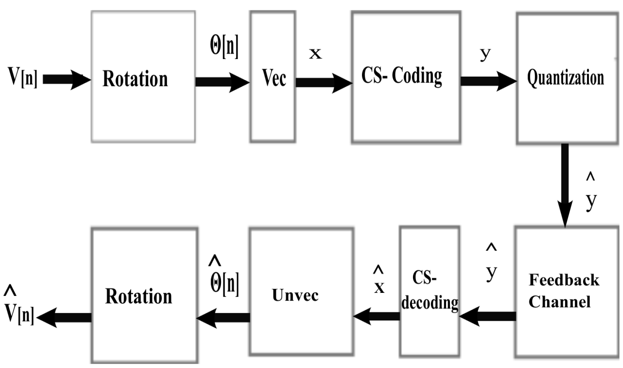

Fig. 1. Algorithm 2

Three methods are compared in the simulations in Fig.3, Fig.2 and Fig.4. The first is the perfect channel state information scenario. The second is using Algorithm 1. The third method is using Algorithm 2.

In Fig.3, and Fig.2, sum rates are compared against signal-to-noise-ratio (SNR); result using both matched filter and MMSE receivers is shown. In the second method, a total feedback bits $B = 10$ are used. On the other hand, the CS method uses half the number of bits, $B = 5$. We can observe that the performance of the CS method is almost equal to that of the method without using CS while saving half the number of bits.

The advantage of CS can also be confirmed from the result in Fig.4, where the bit-error-rate is plotted but in this case the CS and without CS using same number of bits. In this case, we observe that the CS method has a better bit error rate performance. These two figures demonstrate the clear advantage of using CS in feedback of singular vectors in rotative based method.

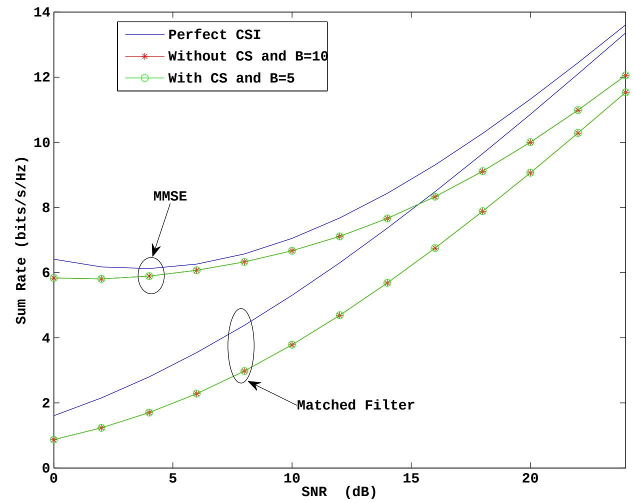

Fig. 2. Sum rate vs. SNR for a 2×2 MIMO system with and without CS with two streams,

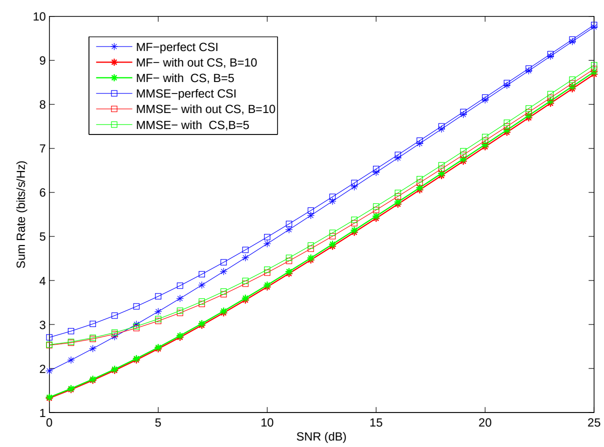

Fig. 3. Sum rate vs. SNR for a 2×2 MIMO system with and without CS with one stream,

VI. Conclusion

In this paper, the concept of compressed sensing (CS) is applied to limited feedback in temporally correlated MIMO channels. The near-sparse nature of the rotation matrices is utilized to combine techniques from CS with rotative quantization for reducing feedback overhead. Simulations show that the use of CS reduces feedback overhead significantly while delivering the same performance. On the other hand, CS based feedback improves performance as compared to direct quantization for the same feedback overhead. CS based limited feedback is in general a promising method that can be applied in various scenarios taking advantage of the sparse nature of the elements to be quantized.

VII. Acknowlegment

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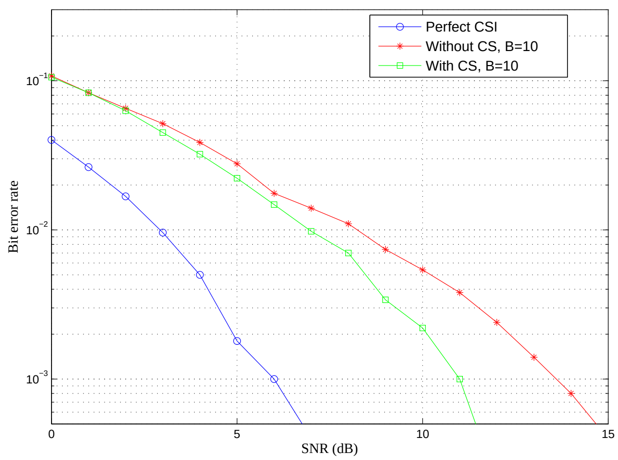

Fig. 4. Bit error rate vs. SNR using matched filter receiver for a 2×2 MIMO system with one stream,

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