Concatenated Zigzag-coded Modulation for Fiber Optical Channels

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***Abstract*—A new coded modulation scheme for ultra-high-speed optical transmission using component concatenated zigzag codes in combination with high-order digital modulation and coherent detection is proposed. In particular, we propose single-level coded modulation and multi-level coded modulation using concatenated zigzag codes, to mitigate the error floor problem associated with the turbo-coded or LDPC-coded systems. For multi-level coded modulation, the mapping bits are divided into several layers and one component concatenated zigzag code is employed at each layer, and the layers are decoded successively in which the decoded layers are used to assist the decoding of the subsequent layers. We provide simulation results to demonstrate that the proposed zigzag-coded modulation system exhibits no error floor at the bit-error-rate (BER) of 10^{-8}, whereas the LDPC-coded system has an error floor around the BER of 10^{-6}.**

I. Introduction

To obtain higher spectral efficiency high-order digital modulation formats have been proposed for high-speed optical transmission systems [1], [2]. Coherent systems are gaining interest with the availability of high-speed signal processing components [3], [4], because they can exploit all optical field parameters in the electrical domain and permit to reach the ultimate limits of spectral efficiency. On the other hand, error control coding based on capacity-approaching channel codes for optical communication systems has received significant attention. Recent works have considered the applications of turbo codes [5], [6] and low-density parity-check(LDPC) codes [7], [8] to fiber optical communications. These codes offer capacity-approaching performance when the codeword length is very long. Due to the very stringent requirement on the residue bit error rate in optical communication systems, the error-correcting codes should exhibit extreme low error floors. Turbo codes suffer from a serious error floor problem in the high signal-to-noise ratio(SNR) range due to the existence of low-weight codewords, especially at high code rates and short packet lengths, which undermines its applicability to high-speed optical systems. A major advantage of the LDPC codes over the turbo codes is the lower error floor. On the other hand, another capacity-approaching error-correcting code, the so-called concatenated zigzag code [9], which also exhibits low error floor. It is of interest to investigate the performance of the zigzag codes in high-speed optical systems that employ high-order modulations and coherent receivers.

In this paper, we propose coded modulation schemes for optical communications using component concatenated zigzag codes in combination with high-order modulation and coherent detection. In single-level coded modulation, only one concatenated zigzag code is employed to encode the information bits and Gray mapping is employed to map the coded bits to channel symbols. In multi-level coded modulation, the mapping bits are divided into several layers and one component concatenated zigzag code is employed at each layer, and the layers are decoded successively in which the decoded layers are used to assist the decoding of the subsequent layers. For the modulation schemes that can be mapped using Gray mapping, e.g., the 4-, 16-, and 64-QAM, the single-level coded modulation is employed because it can approach the channel capacity; for the modulation schemes that cannot be mapped using Gray mapping, e.g., 8-QAM and ring 16-array, the multi-level coded modulation is employed to approach the channel capacity. Our simulation results indicate that for the modulation schemes with Gray mapping, the single-level zigzag-coded modulation with low-complexity decoding performs within 2.5dB of the Shannon limit (for AWGN channel). For the modulation schemes that cannot be mapped using Gray mapping, with the optimized rate allocation, the multi-level zigzag-coded modulation significantly outperforms the single-level coded modulation. Moreover, compared with the LDPC counterparts which exhibit error floors around the BER of 10^{-6}, the proposed zigzag-coded modulation shows no error floor even at the BER of 10^{-8}.

II. Single-level Zigzag-Coded Modulation

Let x_k be the transmitted coded symbol at time k. We consider an effective AWGN channel, e.g., the output of a linear equalizer in a single-carrier dispersive optical channel, or the output of a single-tap equalizer in an optical orthogonal frequency-division multiplexing system [10]. The received signal at time k can be written as $y_k = x_k + \eta$, where $\eta \sim \mathcal{CN}(0, \sigma^2)$ is the white complex Gaussian noise sample.

A. Single-level Coded Modulation

We consider the quadrature amplitude modulations (QAM) shown in Fig. 1(a), i.e., 4-, 16-, and 64-QAMs. We employ

Gray mapping for the M^2-QAM ($M = 2, 4, 8$). For the signal point at $(2i - M + 1, 2j - M + 1)$, $0 \le i, j \le M - 1$, the first-half of the mapping bits are the Gray-M mapping for i, and second-half are the Gray-M mapping for j. The Gray-M mappings of the indices in $\{0, 1, ...M - 1\}$ are shown in Fig. 1(a) along with the M^2-QAM constellations. One

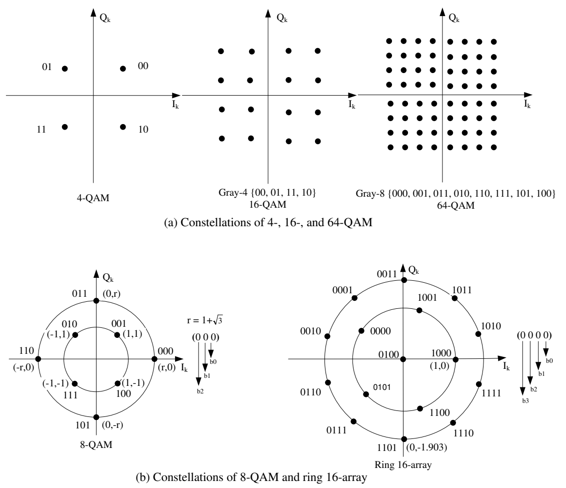

Fig. 1. Mappings for (a) 4-,16-,64-QAM,(b) 8-QAM and ring 16-array.

concatenated zigzag code is employed. The information bits are encoded and then the coded bits are mapped to the QAM symbols according to the mapping rule shown in Fig. 1(a).

B. Single-Stage Demodulation and Decoding

The decoding is done by first computing the LLRs of the coded bits (including the systematic information bits and the parity check bits), and then feeding the LLRs to the zigzag decoder. Since the decoding of zigzag codes has been discussed in [9], we present the LLR computation of each coded bit.

Let $b^B...b^2b^1$ be the mapping bits corresponding to symbol x where $B = \log_2 M^2$ is the number of mapping bits per QAM symbol. Let $A_{i,b}$ be the subset of signal points for which the mapping bit $b^i = b$ for $b = 0, 1$. Given a received signal point y, the LLR for the mapping bit b^i, denoted as $\ell(b^i)$, is

$$\ell\left(b^i\right) = \log \frac{P(b^i = 0|y)}{Pr(b^i = 1|y)} = \log \frac{\sum_{x \in A_{i,0}} P(y|x)}{\sum_{x \in A_{i,1}} P(y|x)}. \qquad (1)$$

III. MULTI-LEVEL ZIGZAG-CODED MODULATION FOR NON-GRAY MAPPING

It is shown in [11] that, for the modulation schemes that can be mapped using Gray mapping, such as 4-, 16-, and 64-QAMs, single-level coding and single-stage decoding can approach the channel capacity. However, for the modulation schemes that cannot be mapped using Gray mapping, such as 8-QAM and ring 16-array in Fig. 1(b), such single-level coded modulation incurs a significant performance gap to the channel capacity. Note that for the ring 16-array, there is one signal point located at the origin; there are 10 signal points on a circle of radius 1.903, i.e., $\left(1.903 \cos \frac{(2k+1)\pi}{10}, 1.903 \sin \frac{(2k+1)\pi}{10}\right)$ for $0 \le k \le 9$; and there are 5 signal points on a circle of radius 1.0, i.e., $\left(\cos \frac{2k\pi}{5}, \sin \frac{2k\pi}{5}\right)$ for $0 \le k \le 4$. To approach the channel capacity, we propose a zigzag-coded modulation scheme based on multi-level coding and multi-stage decoding. Specifically, we divide the B mapping bits into L levels X_1,X_2,...,X_L where $X_i = (b_{i,1}, b_{i,2}, ..., b_{i,\alpha_i})$ for $1 \le i \le L$ and $\sum_{i=1}^{L} \alpha_i = B$. According to the chain rule of mutual information (MI),

$$I(X_1 X_2 ... X_L; Y) = \sum_{i=1}^{L} I(X_i; Y | X_1 X_2 ... X_{i-1}). \qquad (2)$$

The rate corresponding to the left-hand side (LHS) of (2) can be approached by the multi-stage successive decoding corresponding to the right-hand side (RHS), in which at the 1^{st} stage, the bits X_1 are decoded by treating all other bits as random bits; and then at the i^{th} stage the bits X_i are decoded using the decoded bits $\{X_j\}_{0 \le j \le i-1}$ and by treating the bits $\{X_j\}_{i+1 \le j \le L}$ as random bits. In the following we propose a zigzag-coded multi-level coding scheme, where one component code is employed for the mapping bits at each level.

A. Multi-level Zigzag-Coded Modulation

Fig. 2(a) shows the multi-level coding structure consisting of L layers, where each layer is coded by a concatenated zigzag code. An information bit sequence U of length $K = \sum_{i=1}^{L} K_i$ is first demultiplexed into L subsequences $U_1, U_2, ..., U_L$ of lengths $K_1, K_2, ..., K_L$, respectively. The

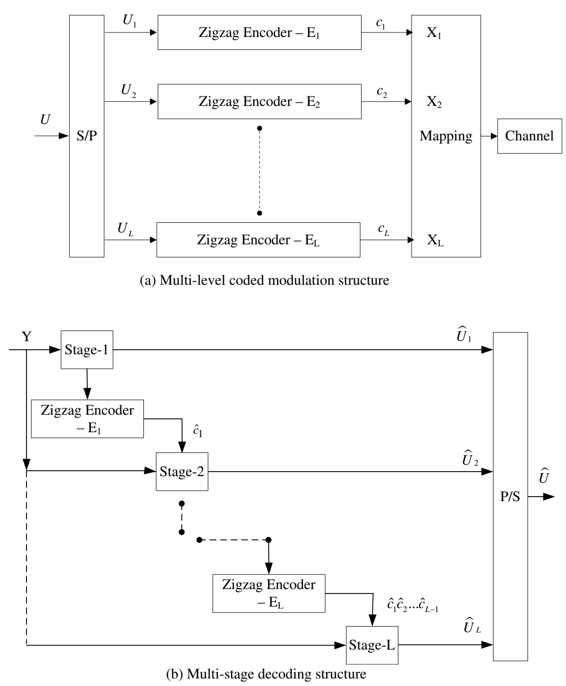

Fig. 2. Multi-level encoding and multi-stage decoding with component zigzag codes.

L subsequences are then encoded by the L concatenated zigzag codes $E_1, E_2, ..., E_L$, respectively, to L codewords $\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_L$ of lengths $N\alpha_1, N\alpha_2, ..., N\alpha_L$, where N is the number of channel symbols that U is encoded and mapped to. For $1 \leq i \leq L$, let $\boldsymbol{c}_i = (c_{i,1}, c_{i,2}, ..., c_{i,N\alpha_i})$.

Mapping Rule: Each channel symbol corresponds to B bits, with α_i bits from layer i, $i = 1, ..., L$. That is, the bit vector corresponding to symbol j, $1 \leq j \leq N$, is given by

$$b^j_{1,1}, ..., b^j_{1,\alpha_1}, b^j_{2,1}, ..., b^j_{2,\alpha_2}, ..., b^j_{L,1}, ..., b^j_{L,\alpha_L}, \quad (3)$$

where $b^j_{i,k} = c_{i,(j-1)\alpha_i+k}$ for $1 \leq k \leq \alpha_i$ and $1 \leq i \leq L$.

Code and Spectrum Rates: The rate of the zigzag code E_i is given by

$$R_i = \frac{K_i}{N\alpha_i}, \quad 1 \leq i \leq L. \quad (4)$$

The spectrum rate (i.e., the number of bits in each channel symbol) of the multi-level coding scheme is

$$R_S = \frac{\sum_{i=1}^L K_i}{N} = \sum_{i=1}^{L} \alpha_i R_i; \quad (5)$$

and the coding rate is given as

$$\bar{R} = \frac{R_S}{\sum_{i=1}^L \alpha_i} = \frac{\sum_{i=1}^L \alpha_i R_i}{\sum_{i=1}^L \alpha_i}. \quad (6)$$

B. Multi-Stage Decoding

Next we describe a multi-stage decoder for the above multi-level coding scheme, and illustrate it by an example of 8-QAM with $L = 2$ layers. The multi-stage decoding structure is shown in Fig. 2(b). When decoding the information bits U_i at stage i, we employ the reencoded codewords $\{\hat{\boldsymbol{c}}_j\}_{1\leq j\leq i-1}$ of the information bits $\{\hat{U}_i\}_{1\leq i\leq j-1}$ decoded in the previous $(i-1)$ stages to compute the LLR of the coded bits of the zigzag code E_i. For the LLR computation of the coded bits, the main idea is that the reencoded mapping bits obtained in the previous stages constrain the possible constellation points to a subset of the entire constellation. At stage i, given the reencoded codewords $\{\hat{\boldsymbol{c}}_j\}_{1\leq j\leq i-1}$, the possible constellation points of the channel symbol j, $1 \leq j \leq N$, corresponding to the coded bits $\{c_{i,(j-1)\alpha_i+m}\}_{1\leq m\leq\alpha_i}$ are constrained to a subset of the entire constellation signal points, denoted as A^i_j. For $1 \leq m \leq \alpha_i$, let $A^i_{j,m,0}$ and $A^i_{j,m,1}$ denote the subset of A^i_j containing the signal points for which the mapping bit $b_{i,m} = 0$ and $b_{i,m} = 1$, respectively. Similarly to (1), the LLR of bit $c_{i,(j-1)\alpha_i+m}$ is given by

$$\ell(c_{i,(j-1)\alpha_i+m}) = \log \frac{\sum_{x\in A^i_{j,m,0}} P(y|x)}{\sum_{x\in A^i_{j,m,1}} P(y|x)}. \quad (7)$$

The multi-stage decoding procedure is summarized as follows.

- For $i = 1$ to L, decode information bits $\hat{U}_i$ of the zigzag code E_i according to the following three steps:
  - Compute the LLR of the coded bits of code E_i based on the reencoded codewords $\{\hat{\boldsymbol{c}}_j\}_{1\leq j\leq i-1}$ and the channel output y according to (7).
  - Decode information bits $\hat{U}_i$ based on the LLR obtained from the first step, using the MLM algorithm described in Section II.B.
  - Reencode information bits $\hat{U}_i$ using the zigzag code E_i to codeword $\hat{\boldsymbol{c}}_i$, which is then used to decode the information bits $\{\hat{U}_j\}_{i+1\leq j\leq L}$ in the substages.
- Multiplex and output the decoded information bits $\{\hat{U}_i\}_{1\leq i\leq L}$.

We next give an example of the multi-level coding scheme and the associated multi-stage decoding for 8-QAM.

1) An Example of 8-QAM: The 8-QAM constellation considered is shown in Fig. 1(b) [10]. We consider a 2-level coded modulation scheme where the mapping bits are divided into two groups $X_1 = (b_0)$ and $X_2 = (b_2 b_1)$. Two concatenated zigzag codes are employed, with the codeword lengths N and $2N$ for the two levels X_1 and X_2, respectively. The information bits stream U are divided into substreams U_1 and U_2, which are then encoded by the two concatenated zigzag codes. The coded bits of the two concatenated zigzag codes are then mapped to 8-QAM symbols using the mapping rule.

The decoding procedure is elaborated as follows.

- In the first stage, since no codeword has been decoded before, the possible set of constellation points is the entire constellation and thus in (7) we have $A^1_{j,k,0} = \{(000), (010), (100), (110)\}$ and $A^1_{j,k,1} = \{(001), (011), (101), (111)\}$. We decode the information bits $\hat{U}_1$ based on the obtained LLRs and then reencode $\hat{U}_1$ to the codeword $\hat{\boldsymbol{c}}_1$, which is used to decode $\hat{U}_2$.
- In the second stage, given the reencoded codeword $\hat{\boldsymbol{c}}_1$ in the first stage, for each transmission symbol the set of constrained signal points $A^2_j = \{(000), (010), (100), (110)\}$ if in the reencoded codeword $\hat{\boldsymbol{c}}_1$ the corresponding mapping bit $b_0 = 0$ and $A^2_j = \{(001), (011), (101), (111)\}$ if in the reencoded codeword $\hat{\boldsymbol{c}}_1$ the corresponding mapping bit $b_0 = 1$. We then decode the information bits $\hat{U}_2$ based on the LLRs obtained in (7).
- Finally, we multiplex $\hat{U}_1$ and $\hat{U}_2$, and output the decoded information bits.

C. Parameter Selection

Finally we give the parameter selection scheme for the proposed zigzag-coded multi-level modulation. The system parameters involved can be classified as the modulation parameters and the code parameters, as follows:

- Modulation parameters: the number of layers L, and the mapping bits $\{X_i\}_{i=1}^L$ for the layers;
- Code parameters: the number of channel symbols N that U is encoded and mapped to, and the parameters (I_i, J_i, K_i) for the concatenated zigzag code employed for layer i, $1 \leq i \leq L$.

Given the average code rate $\bar{R}$ for the zigzag-coded modulation, we propose a parameter selection scheme for the multi-level coded modulation.

1) Modulation Parameter Selection: In theory the capacity-approaching performance can be achieved by having B layers with each layer corresponding to one mapping bit. However, this scheme has a high decoding complexity and delay. It is therefore necessary to consider a scheme which groups the mapping bits into fewer layers.

The capacity-approaching performance of the Gray mapping in the single-stage decoding is due to the fact that any pair of adjacent constellation signal points differ only by one bit. Thus, when grouping the mapping bits, we should put a constraint for any pair of adjacent constellation signal points that *among the bits in the same layer* at most one bit differs, or reduce the number of adjacent constellation signal point pairs for which this constraint is violated. In the following we propose a bit grouping scheme based on the above idea.

For each constellation signal point, there is a shortest Euclidian distance between it and any other signal point. Let d_{th} be the maximum of the such shortest distances among all signal points. Signal points $\boldsymbol{s}_m$ and $\boldsymbol{s}_n$ are *adjacent* if the distance $\|\boldsymbol{s}_m - \boldsymbol{s}_n\| \leq d_{th}$. To see whether the bits b_i and b_j can be grouped into a layer, we examine the following metric showing the number of *bad* adjacent signal pairs for which the bits b_i and b_j are both different,

$$M_{ij} = \sum_{\boldsymbol{s}_m, \boldsymbol{s}_n, \|\boldsymbol{s}_m - \boldsymbol{s}_n\| \leq d_{th}} \mathbf{1}\{b_i(\boldsymbol{s}_m) \neq b_i(\boldsymbol{s}_n), \\ b_j(\boldsymbol{s}_m) \neq b_j(\boldsymbol{s}_n)\}, \quad (8)$$

where $b_i(\boldsymbol{s}_m)$ denotes the bit b_i of the signal point $\boldsymbol{s}_m$ and $\mathbf{1}\{\cdot\}$ is the indicator function. Bits b_i and b_j can be grouped together if $M_{ij} \leq M_{th}$ for some threshold M_{th}, where typically we set $M_{th} = 1$. Once bits b_i and b_j have been grouped, when grouping the remaining bits we set $\|\boldsymbol{s}_m - \boldsymbol{s}_n\| = +\infty$ for the signal points $\boldsymbol{s}_m$ and $\boldsymbol{s}_n$ satisfying $b_i(\boldsymbol{s}_m) \neq b_i(\boldsymbol{s}_n)$ or $b_j(\boldsymbol{s}_m) \neq b_j(\boldsymbol{s}_n)$ because they have been already separated after b_i and b_j are decoded. We run the following bit partition algorithm to obtain the number of layers L and the mapping bits for each layer X_j, $1 \leq j \leq L$.

- Initialize $X = \{b_{B-1}, b_{B-2}, ..., b_0\}$ and $\|\boldsymbol{s}_m - \boldsymbol{s}_n\|$ as the distance between the constellation signal points $\boldsymbol{s}_m$ and $\boldsymbol{s}_n$. Initialize $L = 0$.
- While some mapping bits have not been grouped, do the following
  - Set $L \leftarrow L + 1$. Compute M_{ij} according to (8) for $b_i, b_j \in X$. Let $\mathcal{A}_X$ be the set of all subsets $X' \subseteq X$ such that the *bad pairs* [cf. (8)] of the mappings bits in X' do not exceed M_{th}, i.e.,

    $$X' \in \mathcal{A}_X \Leftrightarrow X' \subseteq X \text{ and } \sum_{b_i, b_j \in X'} M_{ij} \leq M_{th}. \quad (9)$$

    If $\mathcal{A}_X \neq \emptyset$, then we group the mapping bits $X_L = \arg\max_{X' \in \mathcal{A}_X} |X'|$ for layer L, i.e., the element of $\mathcal{A}_X$ with the largest cardinality; otherwise, we set $X_L = \{b_{i_0}\}$ where

    $$i_0 = \arg\max_{i, b_i \in X} \sum_{b_j \in X, j \neq i} M_{ij}, \quad (10)$$

    because $\sum_{b_j \in X, j \neq i} M_{ij}$ *bad signal point pairs* [cf. (8)] can be removed after b_{i_0} is decoded.
  - Update $X \leftarrow X \setminus X_L$; and set $\|\boldsymbol{s}_m - \boldsymbol{s}_n\| = +\infty$ if $b_i(\boldsymbol{s}_m) \neq b_i(\boldsymbol{s}_n)$ for some $b_i \in X_L$, because $\boldsymbol{s}_m$ and $\boldsymbol{s}_n$ can be separated after decoding b_i.

2) Code Parameter Selection: Let $\alpha_j = |X_j|$ for $1 \leq j \leq L$. As the code rate of the concatenated zigzag code is $J/(J+K)$, we construct the following set of discrete code rates

$$\mathcal{R} = \left\{ \frac{J}{J+K}, K \geq 3, \ J + K \leq N_m \right\}, \quad (11)$$

for some maximal value N_m, where $K \geq 3$ is to ensure the capacity-approaching performance of the concatenated zigzag codes [9]. After running the rate allocation procedure, we quantize the allocated rate R_j of each layer j to the closet rate in $\mathcal{R}$, and find the corresponding code parameter (J_j, K_j). We select the parameters I_j, $1 \leq j \leq L$, and N such that the code block lengths

$$I_j(J_j + K_j) = N\alpha_j, \quad 1 \leq j \leq L. \quad (12)$$

IV. Simulation Results

We have performed simulations for the single- and multi-levels modulation schemes using the K-dimensional concatenated zigzag codes, and compared the residue BER of the proposed zigzag-coded modulation to that of the LDPC-coded counterparts. When decoding the concatenated zigzag codes, we set the number of iterations as 20. The concatenated zigzag codes are decoded using the MLM decoding algorithm, and the LDPC codes are decoded by the min-sum decoding algorithm, which is based on the same approximation principle as that of the MLM decoding. In Fig. 3, we plot the BER performance of the single-level coded modulation employing 4-, 16-, and 64-QAM and rate-0.7 concatenated zigzag codes with $(I, J, K) = (6600, 14, 6)$. It is seen that the single-level coded modulation schemes with MLM decoding perform within 2.5dB from the

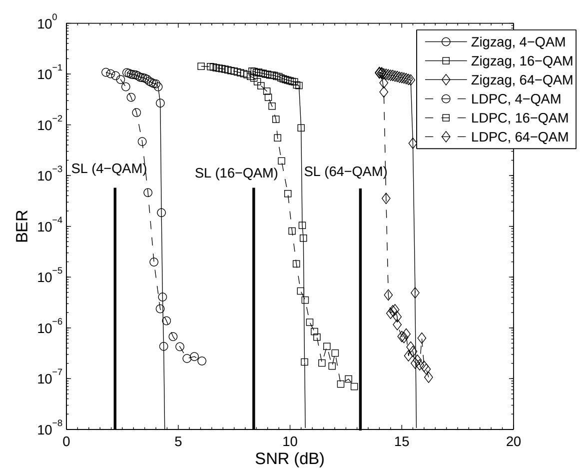

Fig. 3. Comparison of single-level zigzag coded modulation with 4-, 16-, and 64-QAMs.

corresponding Shannon limit marked as "SL". For comparison, we also plot the BER of the irregular LDPC codes optimized using the extrinsic information transfer functions [12]. The proposed system exhibits no error floor at the BER of 10^{-8}, while the LDPC-coded system has an lower error floor around the BER of 10^{-6}. Moreover, the highly regular structure of the zigzag codes, compared with the irregular structure of the optimized LDPC codes, are very attractive from the implementation point of view. Next we illustrate the performance of the multi-level coding and multi-stage decoding. Fig. 4 shows the BER performance of the 8-QAM and ring 16-array [cf. Fig. 1] with optimized rate allocation for the average code rates $\bar{R} = 0.7$ and $\bar{R} = 0.75$, respectively. For the 8-QAM,

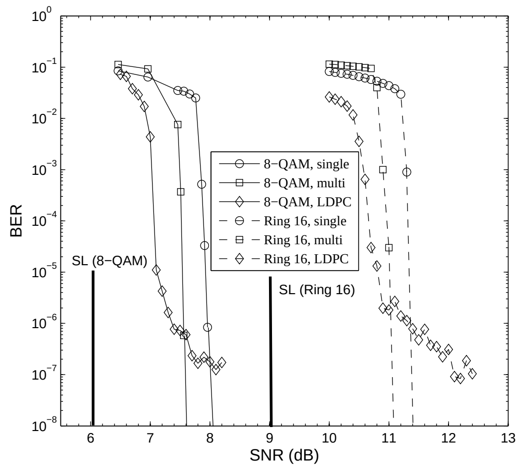

Fig. 4. Performance of multi-level zigzag coded modulation with 8-QAM and ring 16-array.

the mapping bits are grouped into two levels $X_1 = \{b_0\}$ and $X_2 = \{b_1, b_2\}$, and the code rates $R_1 = 0.50$ and $R_2 = 0.80$; the number of coded channel symbols $N = 131070$, and the code parameters are $(I_1, J_1, K_1) = (13107, 5, 5)$ and $(I_2, J_2, K_2) = (13107, 16, 4)$. For the ring 16-array, the mapping bits are grouped into two levels $X_1 = \{b_1, b_2\}$ and $X_2 = \{b_0, b_3\}$, and the code rates $R_1 = 2/3$ and $R_2 = 5/6$; the number of coded symbols $N = 45000$, and the code parameters are $(I_1, J_1, K_1) = (5000, 12, 6)$ and $(I_2, J_2, K_2) = (5000, 15, 3)$. We also plot the BER of the single-level coding with the same average code rates. It is seen that the multi-level coded modulation with multi-stage decoding outperforms the single-level coding by 0.4dB. Furthermore, we replaced the concatenated zigzag codes with the optimized irregular LDPC codes of the same rates, and compared the BER performance. Again the proposed zigzag-coded modulation system exhibits no error floor at the BER of 10^{-8}, while the LDPC-coded system has an lower error floor around the BER of 10^{-6}.

We next analyze the gain of the proposed zigzag-coded modulation in optical communications in terms of the optical signal-to-noise ratio (OSNR) and the total transmission distance.

V. Conclusions

We have proposed new coded modulation schemes for optical communications, based on the capacity-approaching concatenated zigzag codes and high-order modulations. For Gray-mapped modulations, we have proposed a single-level zigzag-coded modulation; and for non-Gray-mapped modulations, we have proposed a multi-level zigzag-coded modulation with multi-stage decoding. The salient features of the proposed zigzag-coded modulation schemes include low-complexity encoding and decoding, highly regular code structure that are amenable to high-speed implementation, and extremely low error floor compared with the low-density parity-check codes, which make them the promising coding and modulation techniques for high-speed fiber optic systems.

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