Generalization of Energy Detector Performance using α-μ Fading Model

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Abstract— **Energy Detection is one of the proposed solutions enabling opportunistic spectrum access. This article revisits the problem of energy detection of an unknown deterministic signal over α-μ generalized fading environments. A closed form series based solution of the probability of detection is derived using one of the canonical forms of the generalized Marcum Q_m function with arbitrary real order. Our new expression represents truly unification and generalization of all previous work covering: Nakagami-m with arbitrary non integer fading severity, Weibull, Gamma distributions in addition to smooth interpolation among them. Moreover, it directly reflects the environment physical parameters and clearly demonstrates their influence.**

Index Terms—Energy Detectors, Blind Detector, Generalized Marcum Q-Function, Opportunistic Spectrum Access.

I. Introduction

Energy Detection is the simplest method that provides moderate performance. It simply measures the energy received on a specific frequency band during an observation interval and declares a white space H_0 if the measured energy is less than a properly set decision threshold. Otherwise, it asserts H_1 . Obviously, increasing the decision threshold simultaneously reduces both false alarm and detection probabilities (P_f and P_d respectively) and vice versa [1][2].

Since the first approximation to the statistics of the energy detector output introduced by Urkowitz [3], several efforts have been exerted to incorporate the effect of different fading models on the evaluation of average probability of detection[4] to [8]. Clearly, utilizing different fading models in conjunction with different integral evaluation approaches resulted in a wide variety of expressions each of which is applicable only to a specific fading model. Additionally, to render their solutions in closed form; authors imposed limitations on system and fading parameters that eventually compromised their effectiveness. Furthermore, results based on series expansions are not guaranteed to converge for all conditions. More importantly, some cases are encountered for which no common distributions including Nakagami-m, seem to adequately fit experimental data [9].

Recently, $\propto$-μ distribution was proposed to explore the nonlinearity of the propagation medium resulting from non-homogeneous diffuse scattering [9][10]. Such phenomenon has been neglected in the derivation of previous fading models. Being a new formulation of the Stacy Generalized Gamma Distribution [11], the proposed distribution includes as special cases, other important distributions such as Gamma, Nakagami-m, and Weibull. It also represents an appropriate generalization of such distributions in addition to interpolation among them [9].

In this paper, we derived a generalized closed form expression describing the effect of fading on the probability of detection providing great flexibility inherited from the α-μ model. Up to the authors' knowledge, the new expression for the first time provides a unified solution bridging all known fading models in addition to interpolation among them. Furthermore, during its derivation, the convergence problem was investigated and conformance to one of the well known convergence theorems is checked [12].

In order to demonstrate both generality and effectiveness of the new expression, different plots of Receiver Operating Characteristics (ROC) representing various known fading situations are produced. Benefitting from the generality of this unification, the effect of various fading and system parameters can now be best demonstrated by simultaneous plots of ROC with different conditions.

The remainder of this article is organized as follows: Blind Energy Detector model and performance is briefly discussed in section II. Section III describes how the α-μ fading model maps to conventional models and how the probability distribution function of the signal to noise ratio is derived. In section IV the new generalized average probability of detection based on the specified fading model is used to study the effect of different model parameters supported with careful explanation. The new expression is further simplified in Section V for the special cases of Nakagami-m and Rayleigh and a consolidating conclusion is given in Section VI. Finally, the derivation of the new expression is given in Appendix A showing its convergence justification.

II. Performance of Energy Detectors

Consider the block diagram shown in Figure 1, where the received signal $x(t)$ is pre-filtered to a bandwidth of W Hz, squared and accumulated over the observation time T Secs before testing against a fixed predefined threshold λ.

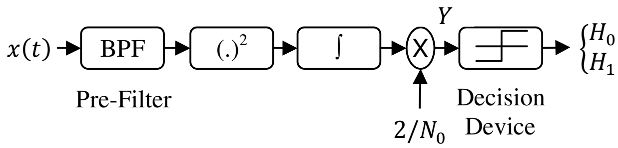

Figure 1: Block Diagram of Energy Detector

Consequently, the receiver decision variable Y takes one of two states depending on signal presence as:

$$x(t) = \begin{cases} n(t) & H_0\ White\ Space \\ h\, s(t) + n(t) & H_1\ Occupied \end{cases} \quad (1)$$

where $s(t)$ is the transmitted signal, $x(t)$ is the received signal, $n(t)$ is the Additive White Gaussian Noise (AWGN) and h is the amplitude gain of the channel [3][4].

Obviously, the received signal envelope is modulated by the random fading amplitude h, having mean-square value $\Omega = \overline{h^2}$ and probability density function (p.d.f) $f_h(h)$. Moreover, the received signal is perturbed by AWGN that is assumed to be statistically independent of the fading amplitude h, and is characterized by a one-sided power spectral density N_0 Watts/Hz. Equivalently, the received instantaneous signal power is modulated by h^2 and consequently, the instantaneous Signal-to-Noise power Ratio (SNR) can be expressed as: $\gamma = |h|^2 \frac{E_s}{N_0}$ with an average $\bar{\gamma} = \Omega \frac{E_s}{N_0}$ where, E_s is the signal energy accumulated over the observation period.

The p.d.f. of γ can be derived by a change of variables as described in [13, 2.3] and is shown as:

$$f_\gamma(\gamma) = \frac{f_h\left(\sqrt{\frac{\Omega\gamma}{\bar{\gamma}}}\right)}{2\sqrt{\frac{\bar{\gamma}}{\Omega\gamma}}} \quad (2).$$

It is now commonly known that the p.d.f. of the decision variable Y can be described by Central and Non-Central Chi Square distributions for H_0 and H_1 respectively with good accuracy. For both distributions the Degree Of Freedom (DOF) of these distributions are shown to be $m = \frac{TW}{2}$ that in general is not restricted to integer values [3]. Consequently, for a fixed threshold λ the conditional probability of false alarm P_f and detection P_d for a certain value of γ can be expressed as [7]:

$$P_f = P\{Y > \lambda | H_0\} = \frac{\Gamma\left(m, \frac{\lambda}{2}\right)}{\Gamma(m)} \quad (3)$$

$$P_d = P\{Y > \lambda | H_1\} = Q_m\left(\sqrt{2\gamma}, \sqrt{\lambda}\right) \quad (4)$$

where, $Q_m(a, b) = \int_0^\infty \frac{x^m}{a^{m-1}} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax)dx$, $I_m(.)$ is the m^{th} order Modified Bessel function [14, 10.25.2] and $\Gamma(a, b)$ is the Incomplete Gamma function [14, 10.25.2].

Fading clearly affects signal to noise ratio γ that in turn affects P_d and hence must be averaged over all of its possible values such that:

$$P_{dav} = \int_0^\infty pd(\gamma)\, f_\gamma(\gamma)\ d\gamma \quad (5).$$

III. Probability of Detection over ∝-μ Fading Channel

A. *Relationship of α-μ to other existing Fading Distributions*

For a fading signal with envelope r, an arbitrary parameter α > 0, and a α-root mean value $\hat{r} = \sqrt[\alpha]{E(r^\alpha)}$, the α-μ probability density function $f_r(r)$ can be written as:

$$f_r(r) = \frac{\propto \mu^\mu}{\Gamma(\mu)} \frac{r^{\propto\mu-1}}{\hat{r}^{\propto\mu}} e^{-\mu\left(\frac{r}{\hat{r}}\right)^\propto} \quad (6).$$

where $\mu > 0$ is the inverse of the normalized variance of r^α, i.e., $\mu = \frac{E^2(R^\alpha)}{V(R^\alpha)}$ and $E(.)$ and $V(.)$ are the expectation and variance operators respectively [9].

Weibull distribution can be obtained from the α-μ distribution by setting $\mu = 1$, and $\alpha = K$. Negative Exponential distribution can result by setting $K = 1$, while Rayleigh distribution can be produced by setting $K = 2$. Furthermore, Nakagami-m distribution can be obtained from the ∝-μ distribution by setting $\alpha = 2$ and $\mu = m$. From the Nakagami-m distribution, setting $m = 1$, again produces Rayleigh distribution and the one-sided Gaussian distribution can be obtained by setting $m = 1/2$. Figure 2 schematically shows the relations of α-μ to other distributions highlighting its apparent flexibility and ability to cover a vast range of fading situations.

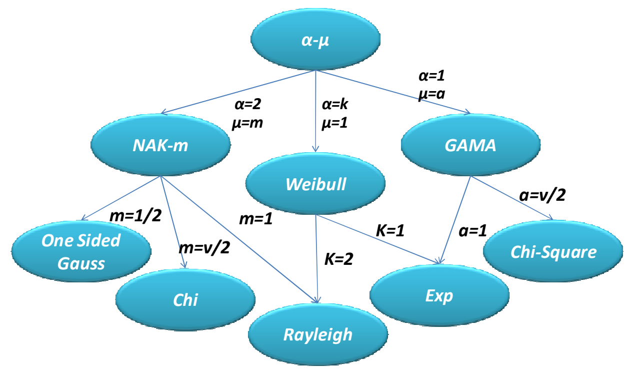

Figure 2: Relations of α-μ to other Distributions

In order to illustrate the effectiveness of utilizing the α-μ fading distribution, the new expression will be plotted using carefully selected parameter sets. The list shown in Table 1 in conjunction with a general non integer parameter m, spans various known fading situations in addition to interpolation among them. For instance, Nakagami-m distribution interpolates between One-Sided Gaussian and Rayleigh while Weibull extends this interpolation to Exponential and finally the Gamma distribution covers the rest of the range.

Table 1: Range of Test Cases

Fading Distribution	α	μ
Exp.	1	1
Weibull (Between Rayleigh-Exp. K=1.5)	1.5	1
Rayleigh	2	1
Nak-m(One Sided Gaussian m=1/2)	2	0.5
Nak-m (Chi m=5)	2	5
Gamma (Chi –Square a=5)	5	1

B. P.D.F. of Signal to Noise Ratio with α-μ Fading Model

Clearly, in the case of the α-μ fading model the envelope distribution $f_r(r;\propto,\mu,\bar{r})$ can be expressed as in (6). Consequently, $f_\gamma(\gamma)$ can be obtained by applying a change of variables in (2) leading to:

$$f_\gamma(\gamma) = \frac{\frac{\propto}{2}\,\mu^\mu}{\Gamma(\mu)} \frac{\gamma^{\frac{\propto}{2}\mu-1}}{\bar{\gamma}^{\frac{\propto}{2}\mu}} e^{-\mu\left(\frac{\gamma}{\bar{\gamma}}\right)^{\frac{\propto}{2}}} \quad (7).$$

The resulting SNR distribution can be observed to follow the α-μ distribution with modified arguments as $f_\gamma\left(\gamma;\frac{\propto}{2},\mu,\bar{\gamma}\right)$ where $\bar{\gamma} = \bar{r}\frac{E_s}{N_0}$.

IV. Probability of Detection over $\propto$-μ Fading Channel

Since, (4) represents the conditional probability of detection given a certain SNR, it is then necessary to average it using (5) with the p.d.f. given in (7).

Following the derivation in Appendix A, the generalized average probability of detection $\bar{P}_d$ will take the form:

$$\bar{P}_d = \frac{\frac{\propto}{2}\,\mu^\mu}{\Gamma(\mu)\ \hat{\gamma}^{\frac{\propto}{2}\mu}} \frac{\sqrt{k}\ l^{\frac{\propto\mu-1}{2}}}{(2\pi)^{\frac{(k+l)}{2}-1}}$$

$$\sum_{n=0}^{\infty} \frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{\Gamma(m+n)} \frac{l^n}{n!} G_{l,k}^{k,l}\left(\left(\frac{\mu}{k\hat{\gamma}^{\frac{\propto}{2}}}\right)^k l^l \,\Big|\, \begin{matrix}\Delta\left(l, 1-\left(n+\frac{\propto\mu}{2}\right)\right)\\ \Delta(k,0)\end{matrix}\right) \quad (8)$$

where, $G_{p,q}^{m,n}\left(x\Big|\begin{matrix}a\\b\end{matrix}\right)$ is the *Meijer-G* function [14,16.17.1], $\Delta(\mathrm{k},\mathrm{a}) = \frac{\mathrm{a}}{\mathrm{k}},\frac{\mathrm{a}+1}{\mathrm{k}}..,\frac{\mathrm{a}+\mathrm{k}-1}{\mathrm{k}}$, l and k are integers such that $\frac{l}{k} = \frac{\propto}{2}$.

One can demonstrate the influence of different fading parameters on the ROC by simultaneously plotting results representing different combinations of these parameters. Figure 3 shows one of such graphs for a sample average SNR of 9 dB and DOF corresponding to $m = 5$ in addition to the AWGN case for comparison.

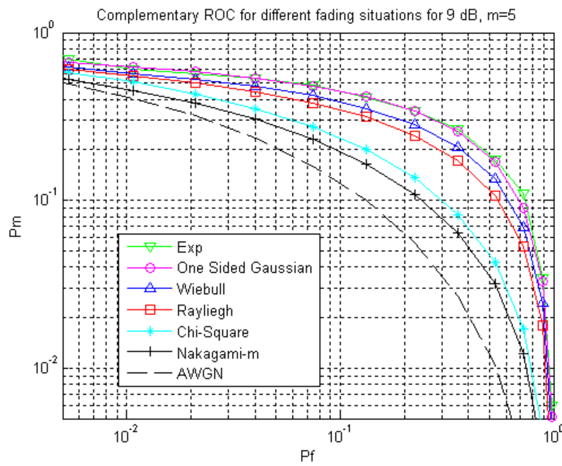

Figure 3: ROC for different fading situations

As expected, the worst case fading occurred with one sided Gaussian corresponding to Nakagami-m severity factor of 0.5 in addition to the exponential distribution. In both cases, lower envelope amplitudes occur more frequent than higher ones. The close match between the two cases in conjunction with being worst cases suggest using Exponential distribution as a performance bound benefiting from its mathematical tractability. On the other hand, situations that can be approximated with Rayleigh and Chi-Square corresponding to lower severity factors clearly show better performance while the best performance is associated with AWGN case.

A. Effect of Environment Non- Linearity parameter α

The nonlinearity parameter α plays an important role in shaping the fading p.d.f. as it solely determines the exponent term. Obviously, increasing α enhances the tail under the p.d.f. and hence for a given fixed threshold it increases the detection probability. This fact is illustrated in Figure 4 showing that increasing this factor considerably enhances the ROC by reducing the Area Under the Curve (AUC).

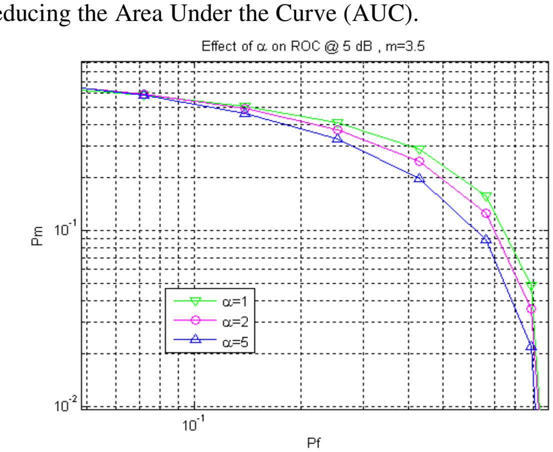

Figure 4: Effect of Non-Linearity Parameter α

B. *Effect of Number of multipath clusters μ*

Physically the parameter μ represents the number of multipath clusters contributing to the received signal envelope. Increasing this factor indicates more diversity in the received multipath clusters and consequently will enhance the probability of detection as shown in Figure 5.

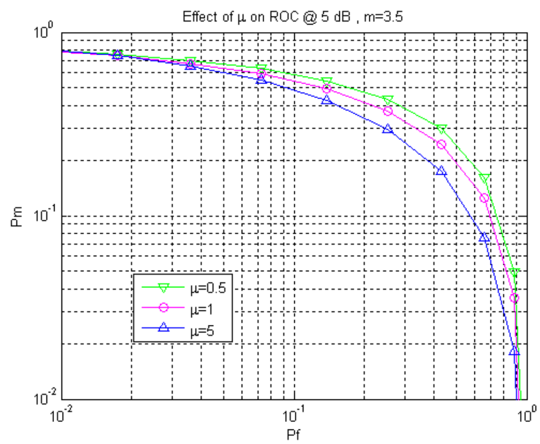

Figure 5: Effect of number of multipath clusters μ

C. *Effect of observation time*

As mentioned earlier, the integration time in conjunction with the filter bandwidth solely determines the DOF of both $H0$ and $H1$ distributions [3]. To shed the light on the effect of this parameter on ROC plots, several plots corresponding to values of m ranging from 1.5 to 10 are shown in Figure 6. Clearly, it has minor effect on ROC and this fact can be best illustrated as follows: For a given bandwidth W, increasing the integration period T is equivalent to increasing m by the same factor. In other words, in reference to [13,4.71] P_f can be rewritten in terms of Marcum Q function as:

$$P_f = Q_m\left(0, \sqrt{\lambda}\right)$$

Now it is clear that m will influence both P_f and P_d by stretching the x-axis of both $H0$, $H1$ p.d.f. Consequently, the pairs P_f and P_d are almost kept unaffected and just occur on different threshold.

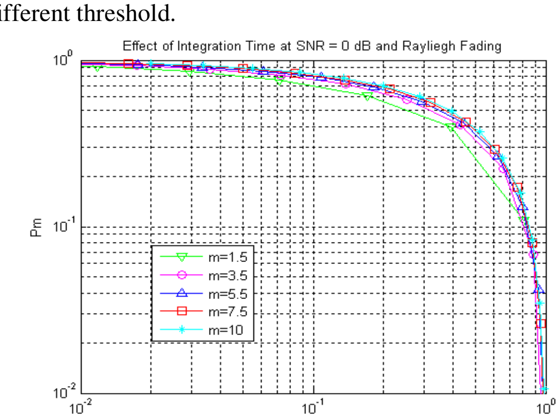

Figure 6: Effect of Integration Time for fixed bandwidth

V. Example of Special Cases

Section III.A indicated that the α-μ distribution reduces to other simpler fading models by setting α and μ to a specific values. For instance it can be reduced to Nakagami-m distribution by letting $\alpha = 2$ with μ taking the value of the Nakagami parameter. Consequently,(8) can be simplified for this case as shown in the following analysis:

$$\bar{P}_{d,NAK} = \left(\frac{\mu^{\mu}}{\Gamma(\mu)\ \ \hat{\gamma}^{\mu}}\right)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{\Gamma(m+n)}\right)G_{1,1}^{1,1}\left(\left(\frac{\mu}{\hat{\gamma}}\right)\middle|\begin{matrix}\Delta(1,1-(n+\mu))\\ \Delta(1,0)\end{matrix}\right)$$

$$\bar{P}_{d,NAK} = \left(\frac{\mu^{\mu}}{\Gamma(\mu)\ \ \hat{\gamma}^{\mu}}\right)\sum_{n=0}^{\infty}\frac{1}{n!}\left(\frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{\Gamma(m+n)}\right)\Gamma(\mu+n)\left(\frac{\hat{\gamma}}{\hat{\gamma}+\mu}\right)^{\mu+n}$$

$$\bar{P}_{d,NAK} = \left(\frac{\mu}{\hat{\gamma}+\mu}\right)^{\mu}\sum_{n=0}^{\infty}\frac{\left(\frac{\hat{\gamma}}{\hat{\gamma}+\mu}\right)^{n}}{n!}\left(\frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{\Gamma(m+n)}\frac{\Gamma(\mu+n)}{\Gamma(\mu)}\right) \quad (9).$$

Eq. (9) can be simplified to (10) using the fact that $\Gamma\left(m+n,\frac{\lambda}{2}\right) = \Gamma(m+n) - G\left(m+n,\frac{\lambda}{2}\right)$ [14, 8.2.3] where $G\left(m+n,\frac{\lambda}{2}\right)$ is the lower incomplete gamma [14, 8.2.1] showing an exact match to the result given in [8].

$$\bar{P}_{d,NAK} = 1 - \sum_{n=0}^{\infty}\frac{1}{n!}\frac{G\left(m+n,\frac{\lambda}{2}\right)}{\Gamma(m+n)}\frac{\hat{\gamma}^{n}\mu^{\mu}}{(\hat{\gamma}+\mu)^{n+\mu}}\frac{\Gamma(\mu+n)}{\Gamma(\mu)} \quad (10).$$

Additionally, as the Rayleigh distribution is a special case of Nagami-m, further simplification to Rayleigh case can be deduced by setting μ=1 to lead to:

$$\bar{P}_{d,RAY} = \left(\frac{1}{\hat{\gamma}+1}\right)\sum_{n=0}^{\infty}\left(\frac{\hat{\gamma}}{\hat{\gamma}+1}\right)^{n}\left(\frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{\Gamma(m+n)}\right) \quad (11).$$

VI. Conclusion

In this work, the performance of Energy Detectors was revisited. A generalized single expression covering a wide range of fading models was derived utilizing α-μ generalized fading model. The new expression relaxed the strict limitation of integer system parameters imposed by previous work in addition to the inclusion of new models like Weibull and Gamma as special cases. Moreover, its effectiveness was demonstrated by plotting several ROC's for the commonly known fading distributions. For the first time, the new expression incorporated the physical environment parameters including nonlinearity and clustering. For fixed average SNR the shape of fading distribution was shown to greatly affect the performance. More fading diversity and nonlinearity factor enhance the AUC while increasing the integration period had minimum influence on this measure.

Appendix A

Evaluation of P_{dav}

Using the canonical form expansion of Marcum Q function in [13, 4.74] combined with [14, 8.4.10] the conditional probability of detection can be shown to take the form:

$$\mathrm{Q_m}\left(\sqrt{2\gamma},\sqrt{\lambda}\right) = e^{-\gamma}\sum_{n=0}^{\infty}\frac{\gamma^n \Gamma\left(m+n,\frac{\lambda}{2}\right)}{n!\,\Gamma(m+n)} \quad (A.1)$$

Substituting (7) and (A.1) in (5) results in:

$$P_{dav} = \int_0^{\infty}\left[e^{-\gamma}\sum_{n=0}^{\infty}\frac{\gamma^n \Gamma\left(m+n,\frac{\lambda}{2}\right)}{n!\,\Gamma(m+n)}\right]\left[\frac{\frac{\propto}{2}\mu^{\mu}\gamma^{\frac{\propto}{2}\mu-1}}{\Gamma(\mu)\hat{\gamma}^{\frac{\propto}{2}\mu}}\mathrm{e}^{-\mu\left(\frac{\gamma}{\hat{\gamma}}\right)^{\frac{\propto}{2}}}\right]d\gamma \quad (A.2)$$

letting, $C = \frac{\frac{\propto}{2}\mu^{\mu}}{\Gamma(\mu)\ \hat{\gamma}^{\frac{\propto}{2}\mu}}$, $u_n(\gamma) = \frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{n!\,\Gamma(m+n)}\gamma^{n+\frac{\propto}{2}\mu-1}\mathrm{e}^{-\mu\left(\frac{\gamma}{\hat{\gamma}}\right)^{\frac{\propto}{2}}}\mathrm{e}^{-\gamma}$, the integral can be put in the following form:

$$P_{dav} = C\int_0^{\infty}\sum_{n=0}^{\infty}u_n(\gamma)\,d\gamma \quad (A.3)$$

Noting that the summand $u_n(\gamma) \geq 0$ over the entire positive real domain and for all $n \geq 0$, then the monotone convergence theorem [12] (*Theorem 5-21)* can be invoked. According to this theorem, term by term integration is permitted if either side converges. This fact can be mathematically stated as:

$$\int_0^{\infty}\sum_{n=0}^{\infty}u_n(\gamma)\,d\gamma = \sum_{n=0}^{\infty}\int_0^{\infty}u_n(\gamma)d\gamma \quad (A.4)$$

In our case the LHS of (A.4) permanently converges to the average probability of detection as it originated from the expansion of Marcum Q function. Consequently, P_{dav} can be expressed as:

$$P_{dav} = C\sum_{n=0}^{\infty}\frac{\Gamma\left(m+n,\frac{\lambda}{2}\right)}{n!\ \Gamma(m+n)}\int_0^{\infty}\gamma^{n+\frac{\propto}{2}\mu-1}\mathrm{e}^{-\mu\left(\frac{\gamma}{\hat{\gamma}}\right)^{\frac{\propto}{2}}}\mathrm{e}^{-\gamma}d\gamma \quad (A.5)$$

It can be observed that the integral $\mathrm{I_n}=\int_0^{\infty}\gamma^{n+\frac{\propto}{2}\mu-1}\mathrm{e}^{-\mu\left(\frac{\gamma}{\hat{\gamma}}\right)^{\frac{\propto}{2}}}\mathrm{e}^{-\gamma}d\gamma$ is just a special case of the Laplace Transform $\mathcal{L}\left\{\gamma^{n+\frac{\propto}{2}\mu-1}\mathrm{e}^{-\mu\left(\frac{\gamma}{\hat{\gamma}}\right)^{\frac{\propto}{2}}}\right\}$ for s =1. As a result and according to [15, 2.2.1-22] if $\frac{\propto}{2}$ is rational number $\frac{l}{k}$ the integral is expressed as:

$$I_n = \frac{\sqrt{k}\ l^{n+\frac{\propto\mu-1}{2}}}{(2\pi)^{\frac{(k+l)}{2}-1}}G_{l,k}^{k,l}\left(\left(\frac{b}{k}\right)^k l^l \middle| \begin{matrix}\Delta(l,-v)\\ \Delta(k,0)\end{matrix}\right) \quad (A.6)$$

where $v = n + \frac{\propto}{2}\mu - 1$, $b = \frac{\mu}{\hat{\gamma}^{\frac{\propto}{2}}}$, $\Delta(k,a) = \frac{a}{k},\frac{a+1}{k}..,\frac{a+k-1}{k}$ and $G_{p,q}^{m,n}\left(x\middle|\begin{matrix}a\\ b\end{matrix}\right)$ is the *Meijer-G* function [14, 16.17.1].

Finally, the generalized form of the average probability of detection is as stated in (8).

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