Optimal Wavelet Basis for Image Compression

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***Abstract*—**Unlike Fourier basis which constitutes fixed sine and cosine *waves;* the Wavelet Transform has infinite basis functions. The choice of good basis is application dependent. Statistical parameters of the image are dynamic and differ from image to image. A moment vector of natural image will be different from the moment vector of synthetic image. Similarly the edges in natural image have structural variations and will be reflected in its subbands whereas synthetic images of thin lines, contours or geometric shapes have least correlation amongst the subbands. Therefore, good basis is a function of image statistical parameters. In this work an effort has been made to implement different classical Orthogonal, Bi-orthogonal and Symmetric wavelets on different images with a view to evaluate good wavelet basis for image compression. This paper discusses the effects of various wavelet functions on different images, zeros and retained energy after thresholding the wavelet coefficients of the decomposed image along with Peak Signal to Noise Ratio of the synthesized image. In order to achieve better compression system, the appropriate wavelet basis are required to be chosen depending upon type of the input image.

***Index terms* —Wavelets, Optimal basis, image compression.**

I. INTRODUCTION

Most of the sensory signals such as still images, video and voice generally contain significant amount of perceptual redundancy in their conical representation with respect to human perceptual system. Compression of Data is employed to decrease the redundancies in data representation and to enhance storage and transmission efficiency. Thus the improvement of good coding techniques will remain to be a design task for future communication systems and in multimedia applications.

Decrease in data redundancy is typically realized by transforming data from one form to another. The popular practices used in the redundancy reduction step are likelihood of the data samples using typical model, transformation of the original data from spatial domain to frequency domain such as DWT or DCT [1-3]. In principle the steps theoretically yields more compact representation of the information in the original data set in terms of fewer coefficients. For lossless image compression, this step is entirely reversible. Transformation of data usually lessens entropy of the original data by removing the redundancies reduction in entropy is attained by dropping non-substantial information in the transformed data established on the application criteria, usually accomplished by quantization technique. Since the entropy of the quantized data is fewer compared to original one, it can be represented by scarcer bits compared to original data set.

Quality measures could be subjective based on human perception or can be objective defined by mathematical or statistical evaluation [4-6]. Zero tree count; monotone spectral ordering across subbands has been used as enactment criteria for wavelet filters in [7]. Although there is no single universally accepted measure of quality metric, yet there are different objective and subjective quality metrics in practice to evaluate data compression algorithms.

This paper formulation is such that section 2 elaborates wavelet transform and overview of existing wavelet based compression techniques. Section 3 discusses the image quality metric used for evaluating experimental results. Section 4 highlights the experimental results obtained by different wavelet basis on various images and its analysis, followed by conclusion in section 5.

II. WAVELET BASED COMPRESSION TECHNIQUES

A. Wavelet Transforms

Wavelet is a small wave whose energy is focused in time and these are functions created from one single function called mother wavelet by dilatations and translations in time domain [89]. If mother wavelet is designated by $\Psi(t)$,the other wavelets $\Psi_{a,b}(t)$ can be signified as

$$\Psi_{a,b} = \frac{1}{\sqrt{|a|}}\Psi\left(\frac{t-b}{a}\right) \tag{1}$$

where *a* and *b* are indiscriminate real numbers and represent dilations and translations respectively. Founded on this definition of wavelets, the wavelet transform of a function f(t) is mathematically represented by

$$W(a,b) = \int_{-\infty}^{\infty}\Psi_{a,b}(t)f(t)dt \tag{2}$$

The inverse transform to recreate f(t) from W(a,b) is mathematically characterized by

$$f(t) = \frac{1}{C}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}W(a,b)\psi_{a,b}(t)dadb \tag{3}$$

where

$$C = \int_{-\infty}^{\infty} \frac{|\Psi(w)|^2}{|w|} dw \quad (4)$$

and W(w) is the Fourier transform of mother wavelet W(t). Since image is treated by a digital computing machine, it is judicious to discretize *a* and *b* and then denote the discrete wavelets accordingly. The most prevalent approach of discretizing a and b is

$$a = a_o^m$$
$$b = nb_o a_o^m \quad (5)$$

where *m* and *n* are integers. Hence DWT can be represented by

$$\psi_{m,n}(t) = a_o^{-m/2}\psi(a_o^{-m}t - nb_o) \quad (6)$$

Wavelets are the footing for representing the images in a hierarchy of growing resolutions, while seeing more and more resolution layers, we get more and more comprehensive look at the image. The remarkable thing about the wavelet decomposition is that it supports zooming feature at absolutely no cost in terms of surplus redundancy.

B. Wavelet Families

Any argument of wavelets activates with Haar, the paramount and simplest [8]. It is discontinuous and looks like a step function. Daubechies are compactly supported orthonormal wavelets thus making the discrete wavelet analysis practicable. Biorthogonal wavelets have linear phase from which many exciting properties are derived. Coiflets wavelets have 2N moments equals zero and the scaling function have the 2N-1 moments equals zero. Both functions have a support of span 6N-1. Symlets are approximately symmetrical wavelets and are similar to Daubechies. Morlet wavelets have no scaling function. Mexican Hat wavelets are derived from a function that is proportional to the second derivative function of the Gaussian probability density function and has no scaling function. Meyer wavelet and scaling function are defined in frequency domain. Other Real wavelets are reverse Biorthogonal, Gaussian derivative family, FIR based approximation of the Meyer wavelet. Some of the complex wavelets are Morlet, Frequency B-Spline and Shanon.

C. Existing Wavelet Based Compression Techniques

Wavelet based compression techniques are primarily focused on thresholding the wavelet coefficients and / or exploiting the correlation with in the subbands. In Embedded Zero Wavelet (EZW) Algorithm, the bits in the bit stream are generated in order of importance, yielding a fully embedded code [10]. EZW consistently produces compression results that are competitive with virtually all known compression algorithms on standard images. Set Partitioning in Hierarchical Trees (SPIHT) provides even better performance than EZW [11]. It is extremely fast and can be made even faster by overlooking entropy coding of the bit stream by arithmetic coding with only a small loss in presentation. Space Frequency Quantization for wavelet Image compression exploits both the frequency and spatial compaction property of the wavelet transform through the use of simple quantization mode[12 13]. The wavelet and subband coding scheme have been used over DCT-based methods such as JPEG, especially at high compression ratios [14-16]. These arrangements enable progressive transmission and browsing. A prioritized quantization scheme can be made for the transform coefficients to attain region dependent quality of coding. In Region of Interest based coding the transform domain image pyramid is subdivided into subpyramids, allowing the straight admittance to image regions, designated regions can then be treated at high fidelity while sacrificing the background providing variable resolution compression. This coding technique can be regarded as modification of what may be called the standard or first generation subband image coder; a combination of a scalar quantizer, subband transformer and an entropy coder. Each of the advances in subband image coding have been succeeded by using the inter band transform domain structure, developing the second generation coding techniques [10 15]. Region based compression can exploit vector quantization technique. An approximate linear system model that can calculate the suitability of candidate filter for compression has been established in [17]. The compression algorithms for digitized images used by Federal Bureau of Investigation are founded on adaptive uniform scalar quantization of discrete wavelet transform subband decomposition, referred as such wavelet/ scalar quantization method [18 19]. Embedded Image Coding method entails three steps, Discrete Wavelet Transform, Binary reduction and Differential Coding. Both A. Said and W. A. Pearlman's and J. Shapiro's embedded zerotree wavelet algorithm code tree algorithm use spatial alignment tree structures to discretely locate the important wavelet transform coefficients [10 11]. Here a direct approach to find the positions of these significant coefficients is presented. The encoding can be clogged at any point, which allows a target rate or distortion metric to be met precisely. The bits in the bit stream are produced in the order of significance, yielding a fully embedded code to successively approximate the original image source, thus well fit for progressive image transmission. The decoder can sack the decoding at any point and yield a lower bit reconstruction image. The regularity and orthogonality of wavelet function is quite favorable in image compression [20]. Although the compactly supported wavelets such as Haar has poor regularity, though it provides superior coding. Number of wavelet based techniques exists for image compression. However, in this paper only the initial step of image decomposition through wavelets has been taken into account and it is open for further exploitation by the existing techniques.

(a) *(b)* *(c)* *(d)* *(e)*

Fig.1. Images used to evaluate objective analysis. (a) Lena (b)Cameraman (c) Building (d)Boy and (e) Shapes.

III. IMAGE QUALITY METRICS

Subjective analysis comprising Mean Opinion Score could not give any meaningful result as significant visual difference using different wavelet basis could not be realized. However, Mean Square Error (MSE) has been calculated by comparing the reconstructed image with original image

$$\mathrm{MSE} = \frac{1}{N}||\mathrm{I} - \mathrm{Ir}||_2^2 = \frac{1}{N}\sum_{i=0}^{N-1}|\mathrm{I_i} - \mathrm{Ir_i}|^2 \qquad (8)$$

where N is the total number of pixels, I is original Image and I_r is reconstructed image. Peak Signal to Noise Ratio (PSNR) has been calculated as

$$\mathrm{PSNR} = 10\log_{10}\frac{255^2}{\mathrm{MSE}} \qquad (9)$$

where *255* is the peak intensity value of the signal.

IV. EXPERIMENTAL RESULTS

Three bench images, a boy image and a synthetic image (Fig.1) have been decomposed by diverse wavelet basis. Wavelet coefficients have been thresholded by a single value and then reconstructed. PSNR, percentage of energy of the reconstructed image and percentage of zeros in the image metric are contained in Table-I. 10% wavelet coefficients have been retained; however effects of retention from 15 to 10% coefficients on PSNR and compression ratio have been reflected in Figure-2. The selection of wavelet basis is forced by linear phase, compact support and perfect reconstruction. The results show that good basis for compression is the function of image statistical parameters. A good wavelet basis for natural image may not be adequate for high frequency images or may be scarce line detector and vice versa. The choice of good wavelet basis can be characterized for image segments instead of complete image. Further that localization is another dilemma. Localization efficiency is impeded by increase of filter coefficients. Haar gives good localization efficiency. It is good detector for lines as well as high frequency contents in image. Similarly Daubechies-4 and Symlets are good for natural images with smooth variations.

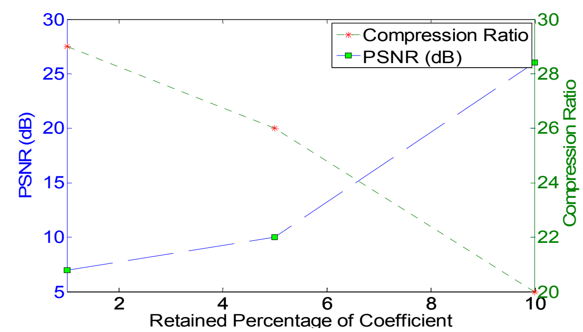

Fig.2. PSNR and Compression Ratio on the percentage of retained coefficient of Lena 256 x 256 Image using Daubechies-4

V. CONCLUSION

This paper addressed the importance of implementing the appropriate wavelet basis for synthetic and natural images. Experiments revealed that shorter the filter length better is its localization efficiency. There is no unique filter that can be termed as optimal. Their performance varies from image to image. The images constituting thin lines, curves, geometric shapes or sudden variations are supported by Haar. Whereas natural images with smooth variation are better coded with lengthy tab Symmetric filters.

REFERENCE

[1] Proc. IEEE (Special Issue on Wavelets), vol. 84, Apr. 1996.

[2] N. Jayant, J. Johnston, and R. Safranek, "Signal compression based on models of human perception," *Proc. IEEE, vol. 81, pp. 1385–1422,* Oct.1993.

[3] Digital Compression and Coding of Continuous Tone Still Images, *ISO/IEC IS 10918*, 1991.

[4] S. Grgic, M. Grgic, and B. Zovko-Cihlar, "Picture quality measurements in wavelet compression system," *Int. Broadcasting Convention Conf.Pub. IBC99, Amsterdam, Netherlands, pp. 554–559, 1999.*

[5] P. C. Cosman, R. M. Gray, and R. A. Olshen, "Evaluating quality of compressed medical images: SNR, subjective rating and diagnostic accuracy," *Proc. IEEE, vol. 82, pp. 920–931*, June 1994.

[6] M. Ardito and M. Visca, "Correlation between objective and subjective measurements for video compressed systems," *SMPTE J., pp. 768–773,* Dec. 1996.

[7] Ramaswamy, V.N.; Rangannathan, N.; Namuduri, K.R.; , "Performance analysis of wavelets in embedded zerotree-based lossless image coding schemes," Signal Processing, IEEE Transactions on , vol.47, no.3, pp.884-889, Mar 1999.

[8] I. Daubechies, Ten Lectures on Wavelets. *Philadelphia, PA: SIAM,* 1992.

[9] S. Mallat, "Multifrequency channel decomposition of images and wavelet models," IEEE Trans. Acoustic., Speech, Signal Processing, vol. 37, pp. 2091–2110, Dec. 1989.

[10] J. M. Shapiro, "Embedded image coding using zero trees of wavelet coefficients," *IEEE Trans. Signal Processing, vol. 41, pp. 3445–3463,* Dec. 1993.

[11] A. Said and W. A. Pearlman, "A new fast and efficient image codec based on set partitioning in hierarchical trees," *IEEE Trans. Circuits Syst. Video Technol., vol. 6, pp. 243–250,* June 1996.

[12] Z. Xiong, K. Ramchandran and M.T. Orchard, "Space-frequency quantization for wavelet image coding," *IEEE Trans. Image Processing,*1997.

[13] Z. Xiong, K. Ramchandran and M.T. Orchard, "Wavelet packets-based image coding using joint space frequency quantization," *Proc ICIP'94, Austin, Texas,* November, 1994

[14] Z. Xiang, K. Ramchandran, M. T. Orchard, and Y. Q. Zhang, "A comparative study of DCT and wavelet-based image coding," *IEEE Trans. Circuits Syst. Video Technology., vol. 9, pp. 692–695*, Apr. 1999.

[15] William B. Pennebaker and Joan L. Mitchel.JPEG Still Image Data Compression Standard.*VanNostrand Reinhold, NY*, 1992.

[16] K. R. Rao and P. Yip, Discrete Cosine Transform: Algorithms,

[17] Advantages and Application, *San Diego, CA: Academic*, 1990.

[18] John D. Villasenor, Benjamin Belzer, and Judy Lio, "Wavelet Filter Evaluation for Image Compression," IEEE Transactions on Image Processing, vol 4, no. 8, August 1995.

[19] Federal Bureau of Investigation. WSQ2 Gray-Scale Fingerprint Image Compression Specification, *IAFIS-IC-0110v2*. Washington DC February, 1993.

[20] Christopher M. Brislawn, Jonathan N. Bradley, Remigius J.

[21] Onyshczak and Ton Hopper. The FBI Compression standards for digitized fingerprint images. *Appl. Digital Image Process XIX, volume 2847 of Proc. SPIE, pp. 344-355, Dnver, CO*, August, 1996.

[22] XiurongBao, "Research of wavelet base's choice in wavelet transformation based on image compression," International Conference on Multimedia Technology (ICMT), 2011, pp.5036-5038, 26-28 July 2011.

TABLE-I

Serial	Wavelet	Average PSNR	Percentages of Energy Retained & Zeros in Image									
			Lena		Cameraman		Building		Boy		Shapes	
1	Haar	33.70	76.69	99.37	85.56	99.37	61.38	99.37	87.55	99.39	91.58	99.99
2	Db 2	33.70	77.70	99.38	86.60	99.38	62.04	99.37	86.08	99.40	90.40	99.70
3	Db 4	33.72	79.60	99.48	86.81	99.52	63.19	99.35	87.81	99.54	90.88	99.98
4	Db 8	33.80	78.39	99.45	86.41	99.51	61.80	99.35	87.92	99.32	89.70	98.06
5	Db 16	33.79	78.51	99.58	85.45	99.58	58.54	99.36	87.45	99.60	87.13	99.96
6	Bior 1.1	33.68	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
7	Bior 1.3	33.66	78.51	99.58	85.45	99.58	58.54	99.36	87.45	99.60	87.13	99.96
8	Bior 1.5	33.55	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
9	Bior 2.2	33.59	76.67	99.35	85.46	99.35	61.16	99.35	87.61	99.38	91.55	99.99
10	Bior 2.4	33.61	79.83	99.49	86.87	99.55	63.11	99.40	81.83	99.57	90.83	99.98
11	Bior 3.5	33.66	76.67	99.35	85.46	99.35	61.16	99.35	87.61	99.38	91.55	99.99
12	Bior 3.7	33.70	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
13	Bior 5.5	33.77	79.50	99.40	85.45	88.35	63.20	99.37	81.00	99.57	90.84	99.97
14	Coif 1	31.96	76.69	99.37	85.56	99.37	61.38	99.37	87.55	99.39	91.58	99.99
15	Coif 2	31.95	77.70	99.38	86.60	99.38	62.04	99.37	86.08	99.40	90.40	99.70
16	Sym 2	33.91	79.60	99.48	86.81	99.52	63.19	99.35	87.81	99.54	90.88	99.98
17	Sym 4	33.89	78.39	99.45	86.41	99.51	61.80	99.35	87.92	99.32	89.70	98.06
18	Sym8	33.88	78.51	99.58	85.45	99.58	58.54	99.36	87.45	99.60	87.13	99.96
19	Rbio 1.1	31.56	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
20	Rbio 6.8	31.49	76.67	99.35	85.46	99.35	61.16	99.35	87.61	99.38	91.55	99.99
21	Mexh	31.25	79.83	99.49	86.87	99.55	63.11	99.40	81.83	99.57	90.83	99.98
22	Morl	31.25	76.67	99.35	85.46	99.35	61.16	99.35	87.61	99.38	91.55	99.99
23	Gaus 1	31.29	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
24	Gaus 2	31.30	79.50	99.40	85.45	88.35	63.20	99.37	81.00	99.57	90.84	99.97
25	Lem 1	30.99	77.70	99.38	86.60	99.38	62.04	99.37	86.08	99.40	90.40	99.70
26	Lem 4	30.87	78.39	99.45	86.41	99.51	61.80	99.35	87.92	99.32	89.70	98.06
27	Cmor 1-1.5	31.10	78.51	99.58	85.45	99.58	58.54	99.36	87.45	99.60	87.13	99.96
28	Cmor 1-1	31.21	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
29	Fbsp 1-1-1	31.22	76.67	99.35	85.46	99.35	61.16	99.35	87.61	99.38	91.55	99.99
30	Fbsp 2-1-0.5	31.25	76.69	99.37	85.61	99.37	61.38	99.37	87.61	99.38	91.58	99.99
31	Shan 1-1.5	31.26	79.81	99.53	86.75	99.58	62.17	99.37	88.75	99.59	89.78	99.96
32	Shan 1-0.5	31.27	76.69	99.37	85.56	99.37	61.38	99.37	87.55	99.39	91.58	99.99