Characteristic Width for Segmentation and Separation of Non-stationary Signals

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Abstract—Segmentation and separation of non-stationary signals is of great interest for many engineering fields and applications. In this paper we present the characteristic width function seen from the joint time-frequency representation of the desired signal. We also propose a segmentation algorithm that is based on the characteristic width of the time-frequency or the dual-frequency distribution of the processed signal. The characteristic function and width is a function that will measure the width of the evolutionary spectrum of non-stationary process. The Time-frequency representations of the signal are obtained using the discrete evolutionary transform DET. The characteristic width function is applied to measure the local energy concentration. Segmentation and separation results give a good measure of the statistical changes due to the frequency changes and identifies the boundary of the these changes on time domain.

Index Terms—Signal segmentation, Characteristics width, Joint Time-frequency Distribution, Discrete Evolutionary Transform.

I. INTRODUCTION

The frequency contents of many signals of applications such as speech, biomedical, seismic and other similar signals evolve with time and their local analysis is of great importance[1][2]. These types of signals are known as non-stationary signals and using standard and regular Fourier Transform is not useful for their analysis. The frequency information who are localized in time as the case of spikes and high frequency bursts cannot be easily detected from the regular Fourier Transform and joint time-frequency analysis becomes the promise analysis tool [3].

Segmentation of multi-components or non-stationary signals has a considerable degree of importance for processing the signals in many applications such as communications, biomedical, and ultrasonic signals. Most of the signal segmentation approaches have been implemented in time domain analysis since the frequency analysis using Fourier transform only reveals spectral information of the processed signal and neglects the time information[4-8]. Time–frequency distribution methods have been employed for non-stationary signal segmentation and separation and that is due to the joint-distribution of the signal’s time and frequency which provide the desired segmentation [9-13].

In this introduction we will continue to introduce the definition of the characteristic width function and the method used to obtain the joint time-frequency kernel. The second part of the paper explains how we developed the segmentation and separation algorithm using the characteristic width function. In the third part, experimental applications with results are provided.

A. Evolutionary Spectrum and the Characteristic Width

Priestley’s Evolutionary Spectrum method [14][15] assumes that the process is oscillatory, i.e., it is composed of sinusoidal components with amplitudes which are slowly varying in time. The process was defined as

\[
x(t) = \int_{-\infty}^{\infty} \phi_n(\omega)dZ(\omega)
\]

where \{Z(\omega)\}, an orthogonal increments process, has the property

\[
E[dZ^*(\gamma)dZ(\gamma')] = \begin{cases} 
0, & \text{when } \gamma \neq \gamma' \\
d_\mu(\gamma), & \text{when } \gamma = \gamma' 
\end{cases}
\]

\(\phi_n(\omega)\) is the family of functions as an amplitude envelope modulating a carrier

\[
\phi_n(\omega) = A_n(\omega)e^{j\beta(\gamma)n}
\]

The carrier frequency \(\beta(\gamma)\) is selected so that the magnitude of the Fourier Transform of the envelope \(A_n(\omega)\), with respect to \(n\), exists and has a maximum at zero frequency. When \(\{x(n)\}\) is stationary process, the family of functions, \(\{\phi_n(\omega)\}\), are the complex exponential, and the expression will reduce to

\[
\phi_n(\omega) = e^{j\omega n}
\]

The energy distribution of the signal jointly over time and frequency is then given by
The oscillatory evolutionary spectrum of the process with respect to the family of function $A_d(w)e^{j\theta}$ was defined as

$$S_{ES}(n,\omega) = |A_n(\omega)|^2$$

The Wold-Cramer representation [16] was defined as

$$x(n) = \sum_{k=0}^{K-1} A(n,\omega_k)e^{j\omega_k n}$$

and coincides with Priestley’s evolutionary spectrum if assume that $H(n,\omega)$ is an oscillatory function. Thus for a non-stationary deterministic signal, or a deterministic signal with a time-dependent spectrum, $x(n)$, $0 \leq n \leq N-1$, an analogous representation is possible [17][18]:

$$x(n) = \sum_{k=0}^{K-1} A(n,\omega_k)e^{j\omega_k n}$$

where $A(n,\omega_k)$ is the time-dependent Gabor kernel obtained using Gabor expansion

$$A(n,\omega_k) = \sum_{l=0}^{N-1} x(l)w(n,l)e^{-j\omega_k l}$$

where $w(n,l)$ is the time-varying Gabor window obtained from the Gaussian function and defined as the Discrete Evolutionary Transform DET [17][18].

The energy density or the evolutionary spectrum is calculated as

$$S(n,\omega_k) = \frac{1}{K}|A(n,\omega_k)|^2$$

and the magnitude of the evolutionary kernel is the energy density in the time-frequency plane and also satisfies the time and frequency marginals.

B. Characteristic Width

According to Priestley’s definition of the evolutionary spectral representation [14], of the $|x(n)|$, is a process whose non-stationary characteristics are changing slowly over time and for each $w$, $A_d(w)$ is, in some sense, a slowly varying function of $n$. A convenient characterization of a slowly varying function is obtained by specifying that its Fourier transform must be highly concentrated in the region of zero frequency. The measure of the width can be done by computing

$$B_{\omega}(\omega) = \sum_{n=0}^{N-1} |\theta| |A_n(\omega)|$$

(1)

where $A_d(w)$ is the time-varying kernel and its Fourier transforms has to be normalized to have a unit integral for each $\omega$.

The characteristic width of the family was defined as the maximum value of the inverse of the width function

$$B_{\omega} = \sup_{\omega} B_{\omega}(\omega)^{-1}$$

(1)

If the process is stationary where the family function is just the complex exponential and does not slowly vary, then the characteristic width is infinite. For the semi-stationary process, the characteristic width is calculated as in the last equation and $2\pi B_{\omega}$ can be interpreted as the maximum interval over which the process may be treated as approximately stationary. Thus the characteristic width is a characterization of the time-dependent spectrum and can be calculated efficiently using our approach shown in the next section.

II. BANDWIDTH ESTIMATION AND SIGNAL SEGMENTATION

A very useful approach of identifying the evolutionary spectrum boundaries (bandwidth) can be obtained from the width function. Instead of the characteristic width, we consider the minimum value of the inverse of the width function which indicates the location of the frequency where the power of the energy is at maximum.

Thus the characteristic width can be redefined as

$$B_{\omega} = \min B_{\omega}(\omega)^{-1}$$

(2)

where $B_{\omega}$ is the width function defined before.

Now for finite deterministic non-stationary signal $x(n)$, the function $|\theta|$ is assume to be a unit function, and then the width function will be reduced to

$$B_{\omega}(\omega) = \frac{1}{K} \sum_{n=0}^{N-1} |X(n,\omega_k)|$$

(3)

Now, by applying the special function defined in (2) which gives the minimum value of the inverse of the width function corresponds to the location of the maximum energy in the frequency domain. Once we identify the frequency location of which the energy is concentrated at, the width of the spectrum can be approximated by a threshold value. The intersection of this threshold with the inverse of the width function gives the boundaries of the evolutionary spectrum of the signal. This approach will provide more details of the energy distribution and will allow measuring the dynamic changes of the signal. The segmentation of any multi-component signal can be achieved using this approach as can be seen in the experimental section.

In order to obtain the desired segmentation, we need to chunk and overlap the processed signal using the time-varying windows $w(n,w)$. Therefore, each overlapped process gives scalar value as the characteristic width. By overlapping and repeating same procedure will provide a number of characteristic width victor of length equal to the number of overlapping chunks or frames.
\[ P(\tau) = \frac{1}{K} \sum_{k=0}^{K-1} |X_n(\tau, \omega_k)|^2 \] 

where \( \tau \) is the time shift or frame.

We notice that the length of the characteristic vector is not equal to the length of the processed signal. Further signal processing is needed at this stage and non-linear interpolation is used to interpolate the data signal to the original signal data points. Due to the non-linearity of the function \( P(\tau) \), a second order Lagrangian interpolation polynomial is used as

\[ f_n(P) = \sum_{j=0}^{n} L_j(P) f(P_j) \]  

where \( n \) stands for the \( n \)th order polynomial and

\[ L_j(P) = \prod_{j \neq i=1}^{n} \frac{P-P_i}{P_j-P_i} \]

is a weighting function that includes a product on \( (n-1) \) terms with terms of \( j=1 \) omitted.

### III. Simulation and Results

To verify the work of our approach, we will consider applying for deterministic non-stationary signal. The expected results must detect and illustrate the segments boundaries in time and frequency domains.

Now let us consider the following multi-component signal \( x(n) \),

\[
x(n) = \begin{cases} 
x_1(n) = \sin(0.4 \cdot \pi \cdot t_1), & t_1 = 1, \ldots, 100 \\
x_2(n) = \sin(0.3 \cdot \pi \cdot t_2), & t_2 = 101, \ldots, 200 \\
x_3(n) = \sin(0.2 \cdot \pi \cdot t_3), & t_3 = 201, \ldots, 300 
\end{cases}
\]

which is composed of three equal segments each with different frequency.

According to our segmentation system, the **DET** is applied to the noisy signal \( x(n) \) in order to obtain its time-dependent spectrum. Notice that we used the overlapping scheme described in section II. The evolutionary spectrum of the signal is shown in Fig. 1. Fig. 2 shows the characteristic width function of the evolutionary spectrum for one single frame of the signal. Additional interpolation was performed to the resulted characteristic function in order to interpolate it to an equal length of the original one. The segmentation result is shown in Fig. 3 where original signal \( x(n) \) is shown at the top and below is the final segmentation vector obtained from our proposed algorithm.

### IV. Conclusions

In this work, we have presented and proposed a practical use of the characteristic function of the evolutionary spectrum for non-stationary signals. The evolutionary spectrum is computed using the discrete evolutionary transform DET. The characteristic width function which measures the bandwidth of the time-dependent spectrum of the signal is also used to give a special measuring function allowing to detect the variation of the signal due to its frequency changes and is the basis for our segmentation approach. In the experimental section, we have applied our algorithm for segmentation of multi-component signal and was successfully able to identify the different segments of the signal.

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Figure 2. Characteristic width function for a single frame.

Figure 3. The multi-component signal and the final segmentation vector.

REFERENCES