Investigation of Time and Frequency Domain Weighting Function for Polyphase Pulse Compression Radars

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***Abstract*—Weighting method by window function is used in order to improve the properties of the code in polyphase pulse compression radars. This paper investigates and compares the effect of weighting methods in frequency domain and amplitude (time domain). Hamming window applied on two common types of polyphase codes; Frank and P3. It is shown that the weighting in frequency domain improves properties of Frank and P3 codes. On the other hand, the amplitude weighting improves properties of P3 code, but in the Frank code, increases the sidelobe level near of the main lobe. Finally, the effect of weighting in frequency and time domain are investigated for Doppler properties of the codes and then the results are compared in detail.**

Keywords-Doppler shift, Hamming window, Polyphase coding, Pulse compression radar, Weighting functions.

I. Introduction

Pulse compression techniques are used in many radar systems for increasing the range of resolution. This is a method combining the high energy of a long pulse with the high resolution of a short pulse. In order to get large time-bandwidth product, the transmitted pulse is modulated by using frequency modulation or phase coding [1].

The matched filter for waveforms pulse compression can be performed by applying the digital correlator. The output of the matched filter will be an extremely narrow pulse with a large peak value, thus the transmitted pulse is compressed in time domain [1]. Unfortunately, the autocorrelation function of a real expanded impulse consists not only of a main peak which is used for target detection but also of range sidelobes which can cover main peaks caused by small targets[1], [2].

Polyphase is one of phase coding methods to overcome this problem. The Frank, P3, and P4 polyphase pulse compression waveforms discussed by the authors [3], [4], provide a class of phase coded waveforms that can be sampled upon reception and processed digitally. These waveforms are derivable from the chirp and step-chirp analog waveforms, so they are similar in certain respects with their analog counterparts. Differences in sidelobe levels, implementation techniques, and Doppler characteristics are some important issues between them.

The compressed pulse of the polyphase coded waveforms has sidelobes which decrease the pulse compression ratio (PCR). For PCR equal to 100, the sidelobe peaks range from 26 to 30 dB below the main peak signal response, depending on the particular code [5].

There are reduction techniques developed to reduce the sidelobe levels. Lewis proposed sliding window two-sample subtractor to reduce the sidelobes for the polyphase codes [6]. In [7], [8] three sidelobe cancellers are developed to reduce peak and integrated sidelobe level. Weighting in frequency and time domain can generally be applied to reduce the sidelobes [5], [9].

In this paper, the effect of weighting in frequency domain is investigated and the Doppler properties of the codes and the results are compared in detail.

This paper is organized as follows. Section II defines the quantities used as performance evaluations and comparison between the considered polyphase codes. Section III describes the classes of polyphase codes. On this basis, results and discussion are given in Section IV. It also illustrates effects of weighting in frequency domain and amplitude weighting on the properties of poly phase codes, and then focuses the effects of weighting on the Doppler performance of the presented codes. Finally, Section V is the conclusion of the paper.

II. Performance Measures

Let N denote the length of each polyphase code $u=\{u(0),u(1),\ldots,u(N-1)\}$. In the sequel, we provide the definitions of the measures used to assist with the performance comparisons of the properties of codes.

A. *Integrated Sidelobe Level*

The integrated sidelobe level ratio (ISL) of a code u of length N indicate the ratio of energy in the sidelobes to that in the mainlobe of the compressed pulse. It is defined as:

$$ISL(dB) = 10\log_{10} \frac{2\sum_{l=1}^{N-1} |C(l)|^2}{|C(0)|^2} \quad (1)$$

where $C(l)$ is amplitude of l th Sample Number of compressed pulse.

B. *Peak Sidelobe Level*

Similarly, the peak sidelobe level (PSL) of a code of length N measures the ratio of the maximum sidelobe magnitude $|C(l)|$ to the inphase value $C(0)$ of the autocorrelation function. It is defined as:

$$PSL(dB) = 20\log_{10} \frac{\max_{1\le l \le N} |C(l)|}{|C(0)|}. \quad (2)$$

III. Classes Of Polyphase Codes

In this section, Frank and P3 codes which are known as polyphase codes will be reviewed. The good properties of these codes in radar application cause them to be used, widely.

A. *Frank Codes*

Elements of this code are arranged to form a $M \times M$ matrix and are given by the M th roots of unity $W = \exp(j2\pi / M)$. The polyphase code of length N is then generated by reading out the matrix of roots of unity row-by-row. This approach has been shown in [10] that the set of polyphase codes of perfect square length $N = M^2$ can be designed for any integer M. The related codes are referred to as Frank codes. The elements $f(m,n)$ of the Frank code in a given set can be arranged as matrix:

$$F = [f(m,n)]_{M\times M} = [\exp(j\frac{2\pi}{M}mn]_{M\times M} \quad (3)$$

and its phases are defined accordingly as:

$$\varphi(m,n) = \frac{2\pi}{M}mn \quad 0 \le m,n \le (M-1) \quad (4)$$

the (m,n) th element can be mapped to the i th element of a code of length N in terms of the phase code as follows:

$$i = mM + n : \varphi(m,n) \rightarrow \varphi(i) = \varphi(mM + n). \quad (5)$$

B. *P3 Code*

P3 code is considered to be cyclically shifted [3]. Accordingly, P3 code is not restricted to perfect square length but can also be constructed for any length N. While the P3 code is shown in [5] to be more Doppler tolerant compared to Frank code.

The phases of P3 code is given by:

$$P3: \quad \varphi(i) = \frac{\pi}{N}(i-1)^2. \quad (6)$$

IV. Results And Discussion

In this section, the effects of weighting in frequency domain and amplitude weighting, using a Hamming window, on the properties of Frank and P3 codes, are investigated. The performance assessment is based on ISL and PSL that reflect code characteristics in the zero Doppler shifts. Block diagram of weighting in frequency and time domain in addition to matched filter based on FFT and IFFT are shown respectively in Fig. 1 and Fig. 2.

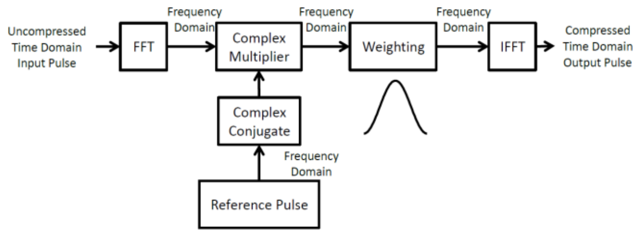

Fig. 1: A block diagram of weighting in frequency domain in addition to matched filter based on FFT and IFFT.

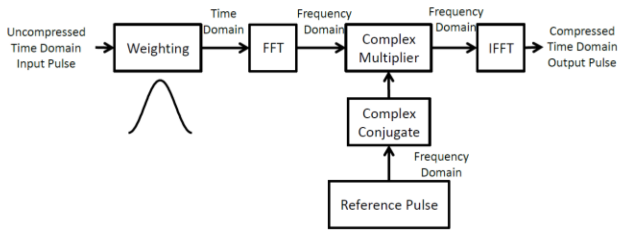

Fig. 2: A block diagram of weighting in time domain in addition to matched filter based on FFT and IFFT.

Also Hamming window can be expressed as follows:

$$w(n) = 0.54 - 0.46\cos(2\pi\frac{n}{N}) \; 0 \le n \le N. \quad (7)$$

At the first step, the performance assessment is based on ISL and PSL that reflect code characteristics in the zero Doppler shifts. At the second step, the behavior in non-zero Doppler shifts is also evaluated.

A. *ISL and PSL in the zero Doppler shift*

Let us first consider ISL and PSL of polyphase pulse compression codes as the performance measures that exhibit the zero-Doppler shift characteristics of the codes.

Fig. 3 shows the compressed pulse and transmitted signal coded with 100-elements Frank code for zero Doppler shifts. As can be seen, PSL and ISL are -29.8 and -13.65 dB, respectively.

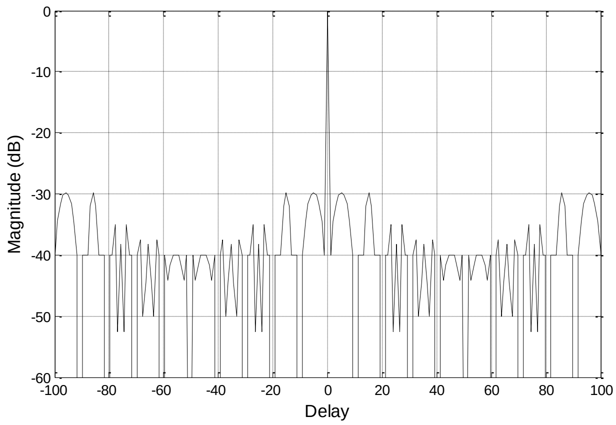

Fig. 3: Compressed pulse of 100-element Frank code in zero Doppler shift.

Applying the weighting in frequency domain changes the code properties, as mentioned. One of the expected changes is the sidelobe level. The effect of weighting in frequency domain for signal, coded with Frank, is shown in Fig. 4. This decreases PSL and ISL to -34.95 and -17.13 dB, respectively.

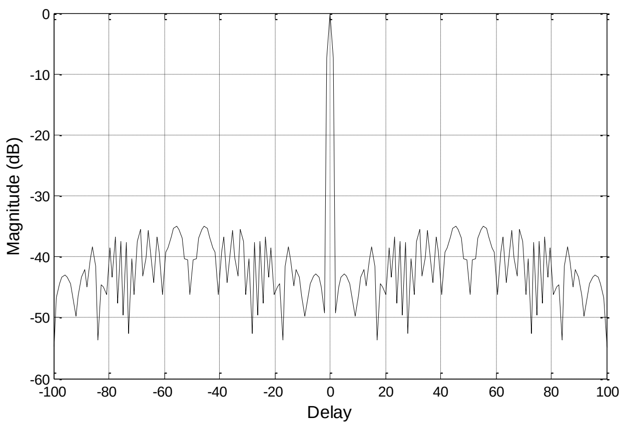

Fig. 4: Compressed pulse of weighted in frequency domain 100-element Frank code in zero Doppler shift.

Also, amplitude weighting of received signal changes the poly phase code properties. This change has positive or negative impact on the code properties. The effect of amplitude weighting for Frank code is shown in Fig. 5. The negative effect of amplitude weighting is obvious in the increase of PSL and ISL to -26.86 and -16.67 dB, respectively.

The results for P3 code shows that the amplitude weighting clearly improves the properties of this code better than weighting in frequency domain. Fig. 6 indicates the compressed pulse for 100 element P3 code. Its PSL and ISL are -26.32, -12.02 dB, respectively. Applying weighting in frequency domain and amplitude weighting are shown in Fig. 7 and Fig. 8, respectively. The weighting in frequency domain decreases PSL and ISL approximately 8.54 and 5.23 dB, respectively. Whereas, using of amplitude weighting decreases the measures, approximately 13.76 and 7.71 dB, respectively.

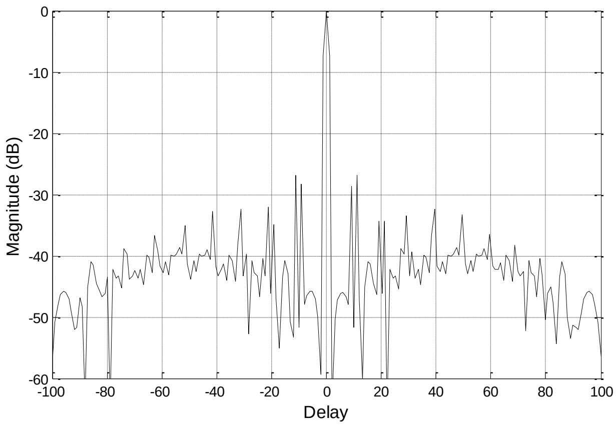

Fig. 5: Compressed pulse of amplitude weighted 100-element Frank code in zero Doppler shift.

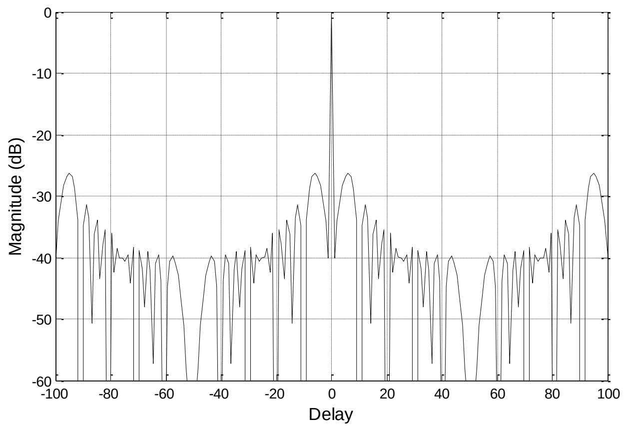

Fig. 6: Compressed pulse of 100-element P3 code in zero Doppler shift.

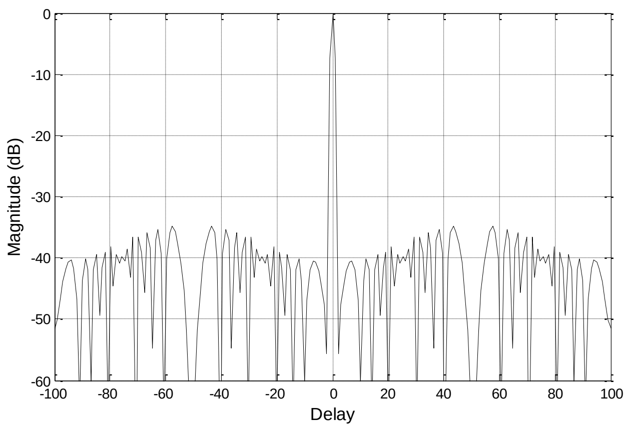

Fig. 7: Compressed pulse of weighted in frequency domain 100-element P3 code in zero Doppler shift.

Weighting in frequency domain and amplitude weighting on Frank and P3 codes show that amplitude weighting improve the properties of P3 code in terms of PSL and ISL, significantly, but this weighting has negative impact on Frank code. As a result properties of Frank code can improve only by the weighting in the frequency domain. Furthermore, the P3 is code that both of weighting methods, improves its properties, but the effect of amplitude weighting is better than the weighting in the frequency domain. All of the results are obtained in zero Doppler shifts. TABLE I summarize results of weighting methods applied to the receive waveform of a 100 element Frank and P3 codes in zero Doppler shift.

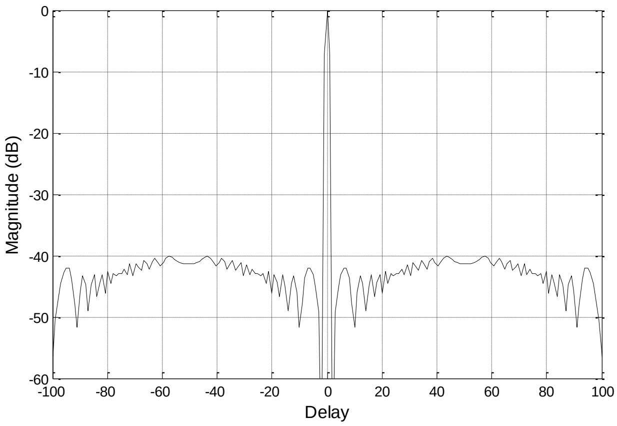

Fig. 8: Compressed pulse of amplitude weighted 100-element P3 code in zero Doppler shift.

TABLE I. Performance for 100 Elements Frank, P3, P4 Codes in Zero Doppler Shift

Polyphase code	Weighting method	Peak Sidelobe Level (dB)	Integrated Sidelobes Level (dB)
Frank	Unweighted	-29.08	-13.65
	Weighting in frequency domain	-34.95	-17.13
	Amplitude weighting	-26.86	-16.67
P3	Unweighted	-26.32	-12.02
	Weighting in frequency domain	-34.86	-17.35
	Amplitude weighting	-40.08	-19.73

B. Comparison of Doppler Responses and Effects of Weighting

In this section, the Doppler responses of the various three 100-element codes are compared by examining the Doppler variation of the peak compressed signal, the peak sidelobes or secondary maxima, and the image signal. These are illustrated in Fig. 9 for a Frank-coded received waveform having a normalized Doppler frequency of -0.05.

In this section, we define two measurement criteria for performance comparison for polyphase codes as follow:

- Ratio of the peak signal to the image lobe.
- Ratio of the peak signal to the secondary maxima.

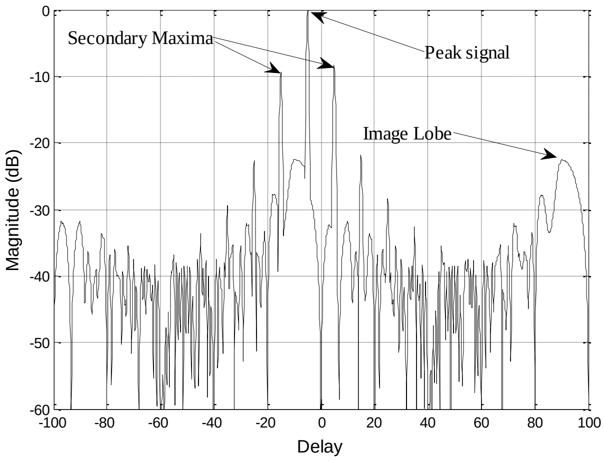

Fig. 9: Compressed pulse of 100-element Frank code, normalized Doppler frequency = - 0.05.

Refer to Fig. 9, the image signal is due to the polyphase codes being derived from a Nyquist rate sampling of the step-chirp or linear chirp phase characteristics. The image lobe has an important role in the sidelobes energy. Also, secondary maxima are near the main peak. The decrease of ratios of the peak signal to the secondary maxima causes the probability of false alarm to increase. Also, secondary maxima can mask main peaks caused by small and relatively weak nearby targets.

Amplitude weighting and weighting in frequency domain bias the main peak, secondary peak and image lobe. The Doppler behavior of the Frank code in three cases: without weighting, amplitude weighting and weighting in frequency domain are shown in Fig. 10. It is obvious that increase of the Doppler shift decreases the main peak, secondary peak and image lobe. In whole cases, the cyclic variation of the peak amplitude is obvious. There is also a cyclic behavior of the secondary maxima and the envelope of the peak signal response every 0.1 in normalized Doppler axis. This Doppler corresponds to a range-Doppler coupling of 10 range cells for the 100-element code, which is the equivalent duration of one phase or frequency group. The peak signal does not fall fast with respect to Doppler shift. Also, amplitude of main peak and secondary peak are double in case one than case two. Both weighting methods, severity decrease the image lobe that is signified reduction of ISL. Weighting in frequency domain reduce the PSL and ISL in payment of the SNR reduction. This note is obvious with attention to changing in the main peak.

Fig. 11 is shown the Doppler behavior of the P3 code in three cases: without weighting, amplitude weighting and weighting in frequency domain. This code is lack of the cyclic variation of the peak amplitude and secondary maxima unlike the Frank code. Image lobe for P3 code increases almost equally and uniformly.

A comparison of the Fig. 10 and Fig. 11 reveals several important observations. First, the percentage cyclic fluctuations of the peak signal, in the weighting methods, decreases in Frank and P3 codes. Second, weighting in frequency domain, decreases signal peak, secondary maxima and image lobe, effectively. The decrease of the signal peak is due to mismatching filter that is undesirable, because it reduces the SNR. Reduction of the SNR in amplitude weighting is much smaller than the weighting in frequency domain. Also, reduction intensity of image lobe and secondary maxima are more than the signal peak. Therefore, weighting in frequency domain, in Frank and P3 codes, increases the ratios of the peak signal to the image lobe and the secondary maxima that is desirable. Third, in whole cases, the secondary maxima in P3 code are generally much less than the Frank code.

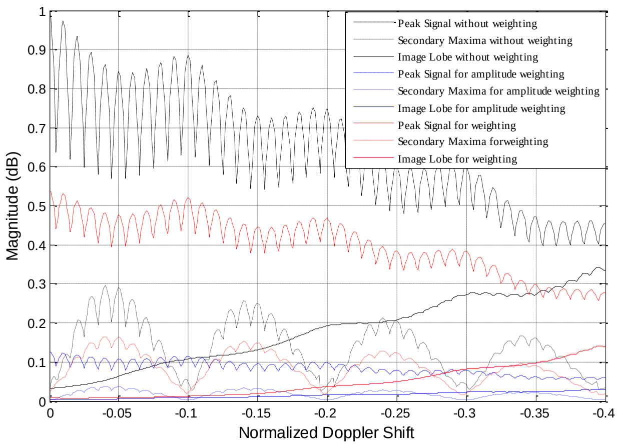

Fig. 10: Doppler properties of 100-element Frank code.

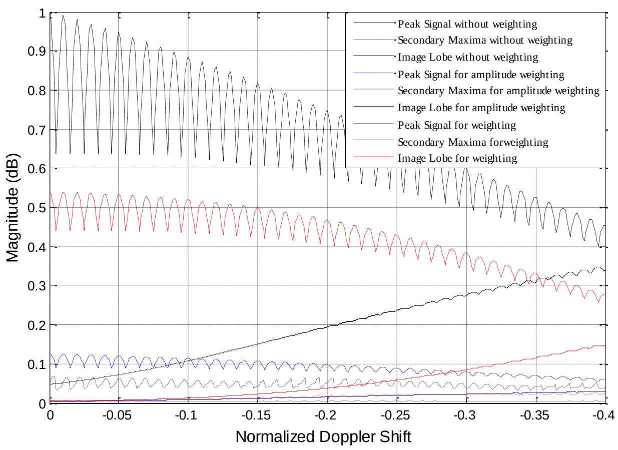

Fig. 11: Doppler properties of 100-element P3 code.

V. CONCLUSIONS

Amplitude weighting and weighting in frequency domain methods are used in radar application in order to reduce the sidelobes level and improve the properties of the code. This paper investigates the effect of these methods on the Frank and P3 polyphase codes. Studies on the Hamming weighting in frequency domain for Frank and P3 codes, in zero Doppler shift, illustrate the improvement of PSL and ISL. Also, amplitude weighting improves the properties of P3 code more effectively than the weighting in the frequency domain, whereas this expression is quite inverse for Frank code. In order to be more accurate, the Doppler behavior of these codes, in non-zero Doppler shift, are compared and studied. Weighting in frequency domain, for Frank and P3 codes, increases the ratios of the peak signal to the image lobe and the secondary maxima. Therefore, amplitude weighting, similar to the weighting in frequency domain, decreases signal peak, secondary maxima and image lobe, strongly. Increase of the ratios of the peak signal to the image lobe and the secondary maxima in P3 code is due to the amplitude weighting. But, this operation on the Frank code increases the ratio of the peak signal to the image lobe and decreases the ratio of the peak signal to the secondary maxima. Finally, the weighting reduces the signal peak and thus reduces the SNR. Of course, reduction of the SNR in amplitude weighting is much smaller than weighting in frequency domain.

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