Orthogonal MPSK (OMPSK) With Bandwidth Efficiency

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Abstract-: Orthogonal MPSK is a coded Multi Phase Shift Keying modulation technique, where the input digital signal is mapped into a block of orthogonal codes. The encoded data, which is in orthogonal space, modulates the carrier frequency by means of MPSK. At the receive side, the data is recovered by means of code correlation. This modulation technique offers channel coding and modulation, with bandwidth efficiency. Construction of rate ½, rate ¾ and rate 1 coded modulation techniques, along with bit error performances, are presented to illustrate the concept.

Index Terms: Error Control, Modulation, Bandwidth.

1. INTRODUCTION

Walsh codes, originally developed by J. L. Walsh in 1923, are well known for their orthogonal properties. These codes have been successfully implemented in CDMA for spreading and user ID. The use of orthogonal codes for forward error control coding has also been investigated by a limited number of authors [1].

Orthogonal Code								Antipodal Code							
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1	1	0	1	0	1	0	1	0
0	0	1	1	0	0	1	1	1	1	0	0	1	1	0	0
0	1	1	0	0	1	1	0	1	0	0	1	1	0	0	1
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0
0	1	0	1	1	0	1	0	1	0	1	0	0	1	0	1
0	0	1	1	1	1	0	0	1	1	0	0	0	0	1	1
0	1	1	0	1	0	0	1	1	0	0	1	0	1	1	0

Figure 1: 8-bit bi-orthogonal code set. It has 8 orthogonal codes and 8 antipodal codes, for a total of 16 bi-orthogonal codes.

Orthogonal codes are binary valued and have equal numbers of 1's and 0's. Antipodal codes, on the other hand, are just the inverse of orthogonal codes. Antipodal codes are also orthogonal among them. Therefore, an n-bit orthogonal code has n-orthogonal codes and n-antipodal codes, for a total of 2n bi-orthogonal codes. For example an 8-bit orthogonal code has 16 bi-orthogonal codes as shown in Figure 1. Since there are equal number of 1's and 0's, each orthogonal code will generate a zero parity bit. Antipodal codes are also orthogonal among themselves. Therefore, each antipodal code will generate a zero parity bit as well.

This paper presents a multiphase shift keying (MPSK) modulation technique using orthogonal codes. This is defined as Orthogonal MPSK (OMPSK).

OMPSK is a coded modulation technique that utilizes a block of bi-orthogonal code to map a block of data. In this scheme, when a block of data needs to be transmitted, the corresponding block of bi-orthogonal code is transmitted by means of MPSK. Upon receiving, the receiver synchronizes, detects and corrects errors by means of code correlation. The proposed coded modulation technique offers synchronization and error control coding in the same platform. The detailed design and bit error performance are presented to illustrate the concept.

2. Encoding Scheme

2.1. Rate 1/2 Orthogonal Coded Modulation

A rate 1/2 orthogonal coded modulation with n = 8 can be constructed by inverse multiplexing the incoming traffic, Rb (b/s), into 4-parallel streams (k = 4) as shown in figure2. These bit streams, now reduced in speed to Rb/4 (b/s) are used to address sixteen 8-bit biorthogonal codes, stored in an 8 x 16 ROM. The output of each ROM is a unique 8-bit orthogonal code, which is then modulated by a QPSK modulator and transmitted through a channel.

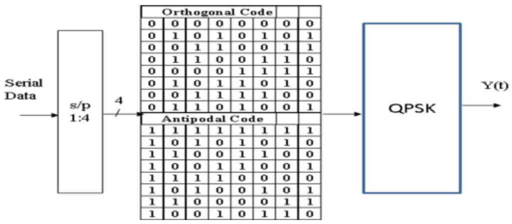


Figure 2: Rate 1/2 orthogonal coded modulation with n = 8.

Since the signal stream is in orthogonal space, it can be expressed as:

$$Y(t) = c_i(t)s_i(t) \tag{1}$$

where $s_i(t) = A(t)\cos[(\omega_c t + \phi_i(t)]$ is the modulating signal and Ci is the respective orthogonal code. The transmission bandwidth is given by

$$bw \approx (1+\alpha)\frac{8}{4}R_b Hz \tag{2}$$

where 0 < α < 1, α being the roll-off factor due to raised cosine filter which is not shown in the figure. The code rate can be readily obtained as r = 4/8 = 1/2.

2.2. Rate 3/4 Orthogonal Coded Modulation

A rate 3/4 orthogonal coded modulation with n = 8 can be constructed by inverse multiplexing the incoming traffic, Rb (b/s), into 6-parallel streams (k = 6) as shown in figure 3. These bit streams, now reduced in speed to $R_b/6$ (b/s), are partitioned into two sub-sets, 3-bits per sub set. Each 3-bit sub-set is used to address eight 8-bit orthogonal codes. These codes are stored in two 8 x 8 ROMs. The output of each ROM is a unique 8-bit orthogonal code, which is then modulated by a QPSK modulator.

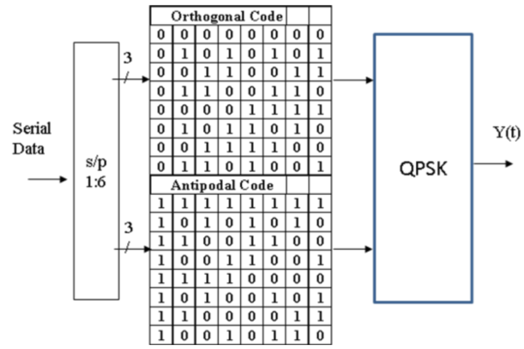


Figure 3: Rate 3/4 orthogonal coded modulation with n = 8.

Since each signal stream is now in orthogonal space, they can be expressed as a linear combination of two non-interfering signals,

$$Y(t) = \sum_{i=1}^{2} c_i(t)s_i(t) \tag{3}$$

where $s_i(t) = A(t)\cos[(\omega_c t + \phi_i(t)]$ is the modulating signal and ci is the respective orthogonal code. The transmission bandwidth is given by

$$bw \approx (1+\alpha)\frac{8}{6}R_b Hz \tag{4}$$

The code rate is given by r = 6/8 = 3/4.

2.3. Rate 1 Orthogonal Coded Modulation

A rate 1 orthogonal coded modulation with n = 8 can be constructed by inverse multiplexing the incoming traffic, R_b (b/s), into 8-parallel streams (k = 8) as shown in figure 4. These bit streams, now reduced in speed to $R_b/8$ (b/s), are partitioned into four sub-sets, two bits per sub set. Each 2-bit sub-set is assigned to address four 8-bit orthogonal codes. The orthogonal codes are stored in four 8 x 4 ROMs. The output of each ROM is a unique 8-bit orthogonal code, which is then modulated by 16-PSK modulator. Since each signal stream is now in orthogonal space, they can be expressed as a linear combination of four non-interfering signals, we write:

$$Y(t) = \sum_{i=1}^{4} c_i(t)s_i(t) \tag{5}$$

where $s_i(t) = A(t)\cos[(\omega_c t + \phi_i(t)]$ is the modulating signal and ci is the respective orthogonal code. The transmission bandwidth is given by

$$bw \approx (1+\alpha)\frac{8}{8}R_b Hz \qquad (6)$$

The code rate is r = 8/8 = 1.

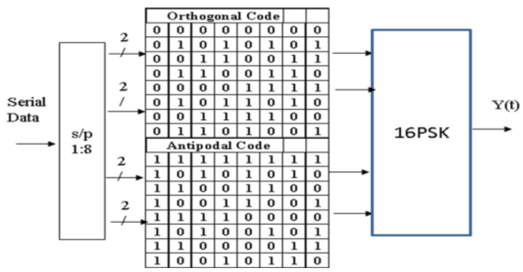


Figure 4: Rate 1 orthogonal coded modulation with n = 8.

3. Decoding Scheme

At the receiver, the incoming impaired orthogonal code is first examined by generating a parity bit. If the parity bit is one, the received code is said to be in error. The impaired received code is then compared to a lookup table for a possible match. Once the closest approximation is achieved, the corresponding data is outputted from the lookup table. A brief description of the decoding principle is given below:

An n-bit orthogonal code has n/2 1s and n/2 0s; i.e., there are n/2 positions where 1s and 0s differ. Therefore, the distance between two orthogonal codes is d = n/2. This distance property can be used to detect an impaired received code by setting a threshold midway between two orthogonal codes as shown in Figure 5, where the received coded is shown as a dotted line. This is given by:

$$d_{th} = \frac{n}{4} \qquad (7)$$

Where n is the code length and dth is the threshold, which is midway between two valid orthogonal codes. Therefore, for the given 8-bit orthogonal code, we have dth = 8/4 = 2. This mechanism offers a decision process, where the incoming impaired orthogonal code is examined for correlation with the neighboring codes for a possible match.

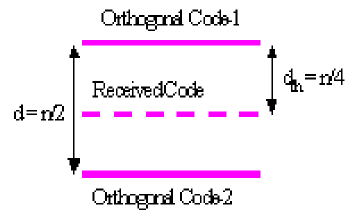


Figure 5: Decoding principle.

The received code is examined for correlation with the neighboring codes for a possible match.

The acceptance criterion for a valid code is that an n-bit comparison must yield a good auto-correlation value; otherwise, a false detection will occur. The following correlation process governs this where an impaired orthogonal code is compared with a pair of n-bit orthogonal codes to yield,

$$R(x,y) = \sum_{i=1}^{n} x_i y_i \geq (n - d_{th}) + 1 \qquad (8)$$

Where R(x,y) is the auto correlation function, n is the code length, dth is the threshold as defined earlier. Since the threshold (dth) is in the midway between two valid codes. An additional 1-bit offset is added to equation-4 for reliable detection. The average number of errors that can be corrected by means of this process can be estimated by combining equations 3 and 4, yielding,

$$t = n - R(x,y) = \frac{n}{4} - 1 \qquad (9)$$

Where t is the number of errors that can be corrected by means of an n-bit orthogonal code. For example, a single error-correcting orthogonal code can be constructed by means of an 8-bit orthogonal code (n = 8). Similarly, a three-error-correcting, orthogonal code can be constructed by means of a 16-bit orthogonal code (n = 16), and so on. Table-1 below shows a few orthogonal codes and the corresponding error- correcting capabilities.

Table1.nOrthogonal Codes and the Corresponding Error Correcting Capabilities

n	t
8	1
16	3
32	7
64	15

4. Coding Gain

We have established that an n-bit orthogonal code can correct t errors where t = n/4-1. A measure of coding gain is then obtained by comparing the word error with coding, Pe(WEC), to the word error without coding, Pe(WEU). We examine this by means of the following analytical means [7]:

Let S =Transmit power and T= Symbol duration. Then the coded and uncoded symbol energy will be,

ST/n = Coded symbol energy (10)
ST/k = Uncoded symbol energy
With coherent PSK modulations, the coded and uncoded symbol error rates will be

$$Q_c = \frac{1}{2} erfc\left(\sqrt{\frac{ST}{nNo}}\right) \quad (11)$$

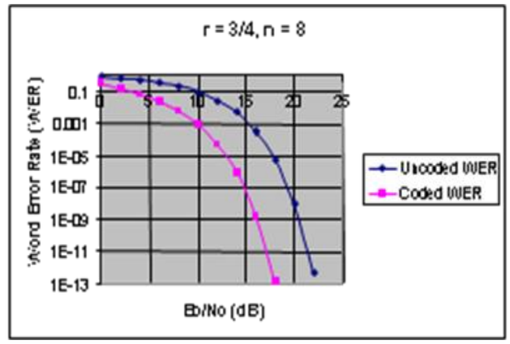
$$Q_u = \frac{1}{2} erfc\left(\sqrt{\frac{ST}{kN_o}}\right) \quad (12)$$

The corresponding word error rates are given by,

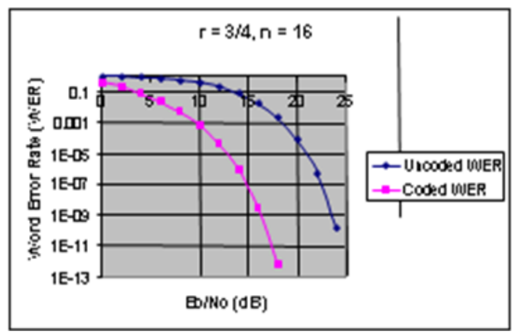
$$\text{Pe(WEC)} = \sum_{i=t+1}^{n} \binom{n}{i} Q^i{}_c (1 - Q_c)^{n-i} \quad (13)$$

$$\text{Pe(WEU)} = 1 - (1 - Q_u)^k \quad (14)$$

Where Qc and Qu are coded and uncoded symbol error rates and Pe(WEC) and Pe(WEU) are the respective word error rates. Figure 6 shows the error performance for several code lengths. As expected, the net gain in word error rate due to coding is evident. We also note that coding gain increases for longer codes. From these results, we conclude that orthogonal codes offer coding gain with bandwidth efficiency.



(a)



(b)

Figure 6: Error performance: (a) r=3/4, n = 8 and (b) r = 3/4, n = 16.

5. Conclusions

We have examined error control properties of orthogonal codes and have shown that the distance properties along with parity can be used to detect and correct errors to protect digital information from impairments. The proposed technique offers a simpler solution to error control coding and bandwidth efficiency. Construction of rate ½ and rate ¾ orthogonal coded modulation schemes along with WER performance analysis indicates that orthogonal codes offer error control coding with bandwidth efficiency.

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